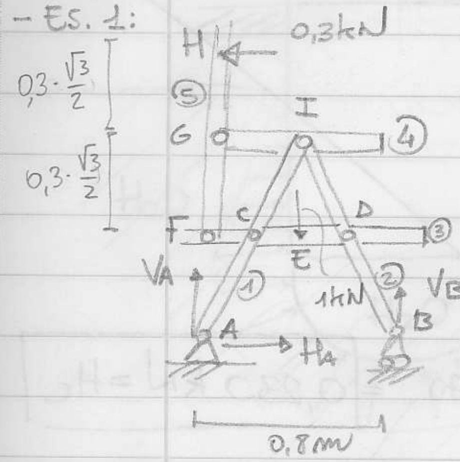
– CORREZIONE COMPITO TCM ENERGETICI 11/06/2009

– ES. 1:



$CE = ED = 0{,}15\ m$

$FC = 0{,}25\ m$

REAZ. VINC:

$V_A + V_B = F_E = 1\,kN$

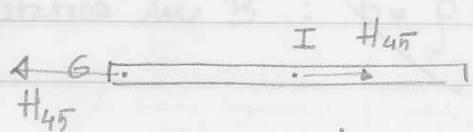
$H_A = F_H = 0{,}3\,kN$

$V_B\cdot 0{,}8 = F_E\cdot 0{,}4 - F_H\cdot\left(0{,}8\cdot\frac{\sqrt3}{2} + 0{,}3\frac{\sqrt3}{2}\right)$

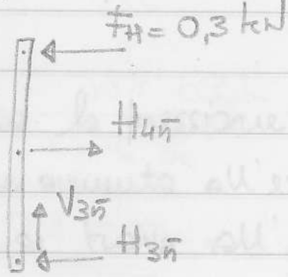
$\Rightarrow V_B = \frac{0{,}4 - 0{,}3\cdot(0{,}95)}{0{,}8} = \underline{0{,}142\ kN}$

$\Rightarrow \underline{V_A = 0{,}858\ kN}$

* CORPO 4



* CORPO 5

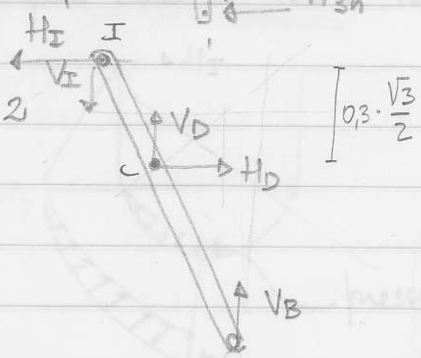


$\Rightarrow V_{35} = 0$

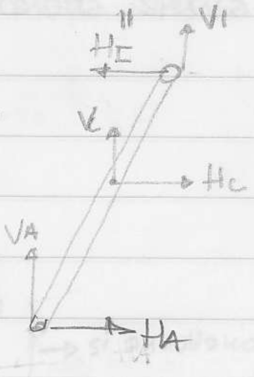
$\Rightarrow H_{45} = 2F_H = 0{,}6\ kN$

$\Rightarrow H_{35} = 0{,}3\ kN$

* CORPO 2

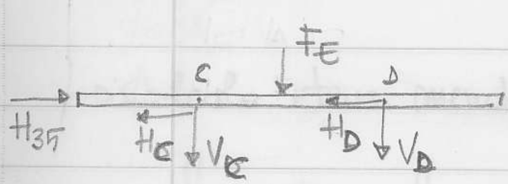


* CORPO 1



$H'_I + H''_I = H_{45} = 0{,}6\ kN$

* CORPO 3



$\Rightarrow$ Eq. mom C

$-F_E\cdot 0{,}15 - V_D\cdot 0{,}3 = 0$

$\Rightarrow V_D = -\frac{F_E}{2} = \boxed{-0{,}5\ kN = V_D}$

$-V_D - F_E - V_C = 0 \Rightarrow V_C = -F_E - V_D = -1 + 0{,}5 = \boxed{-0{,}5\ kN = V_C}$

Eq. mom. corpo 2 risp. I

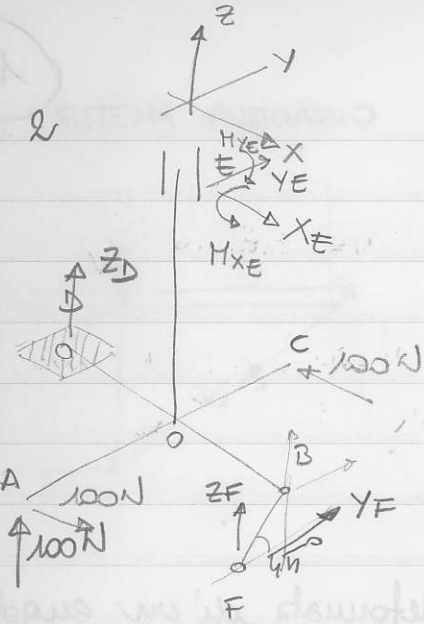
$-H_C - H_D + H_{35} = 0$

$-V_D\cdot 0{,}15 + V_B\cdot 0{,}4 + H_D\cdot 0{,}3\frac{\sqrt3}{2} = 0 \Rightarrow \boxed{H_D = 0{,}07\ kN}$

$\boxed{H_C = 0{,}23\ kN}$

2

ESERCIZIO 2



Determin. reazioni vincolari

$\Leftarrow Z_F = Y_F$ (1° eq)

Eq. lungo z:

$$100 + Z_F + Z_D = 0$$

lungo x

$$X_E = 0$$

lungo y

$$Y_F + Y_E = 0$$

Rot. int x (polo O)

$$-100 \times 1 - Y_E \cdot 1 + M_{XE} = 0$$

Rot. int y (polo O)

$$+ Z_D \cdot 1 - Z_F \cdot 1 + X_E \cdot 1 + M_{yE} = 0$$

Rot. intorno z (polo O)

$$200 \times 1 + Y_F \cdot 1 = 0$$

$\Rightarrow$ $Y_F = -200$ N $\qquad M_{XE} = 300$ Nm

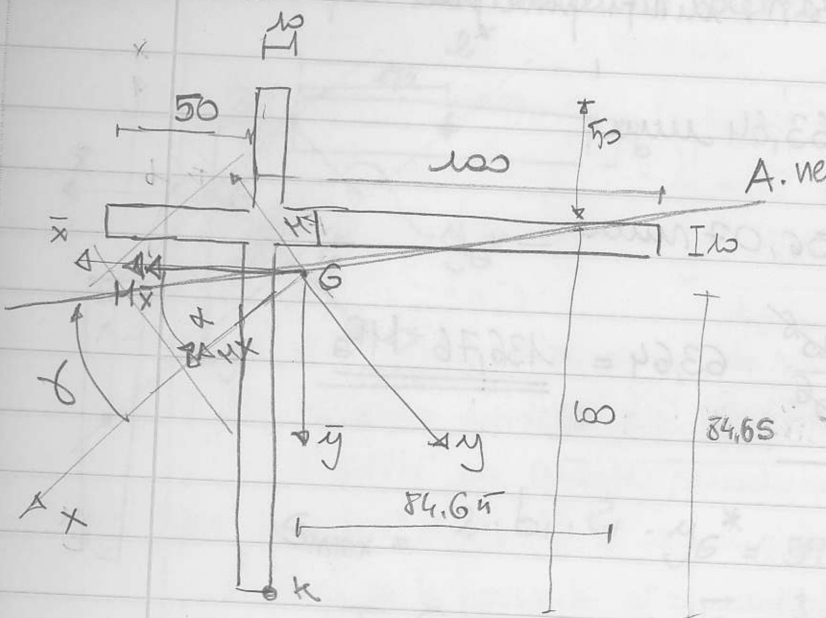
$Z_F = -200$ N $\qquad M_{yE} = -300$ Nm

$Y_E = 200$ N

$X_E = 0$ N

$Z_D = 100$ N

ESERCIZIO 3 ③



$T_{\bar{x}} = 3kN$

$T_{\bar{y}} = -3kN$

$J_{\bar{x}} = J_{\bar{y}} = 3{,}11 \cdot 10^6 \ mm^4$

$J_{\bar{x}\bar{y}} = 3{,}1 \cdot 10^5 \ mm^4$

→ Assi principali: da questioni di simm., $\underline{\underline{\alpha = 45°}}$, x e y nuovi assi principali

⇒ Nuovi valori dei mom. d'inerzia rispetto gli assi principali

$$J_x = J_{\bar{x}} \cdot \cos^2\alpha - J_{\bar{x}\bar{y}} \sin\alpha\cos\alpha + J_{\bar{y}} \sin^2\alpha = 2{,}8 \cdot 10^6 \ mm^4 = J_x$$
$$J_y = J_{\bar{y}} \cos^2\alpha + J_{\bar{x}\bar{y}} \sin\alpha\cos\alpha + J_{\bar{x}} \sin^2\alpha = \underline{3{,}42 \cdot 10^6 \ mm^4 = J_y}$$

FLESSIONE: Ho $M_{\bar{x}} = 5kNm$;

$$M_x = M_{\bar{x}} \cdot \cos\alpha = 3{,}54 \ kNm$$
$$M_y = -M_x \cdot \sin\alpha = -3{,}54 \ kNm$$

ASSE NEUTRO: $0 = \frac{M_x}{J_x} \cdot y - \frac{M_y}{J_y} \cdot x$

$$y = \frac{M_y}{M_x} \cdot \frac{J_x}{J_y} = -1 \cdot \frac{2{,}8 \cdot 10^6}{3{,}42 \cdot 10^6} = \boxed{-0{,}819x = y} \Leftarrow \text{eq. asse neutro}$$

$$-\gamma = \operatorname{atan}(-0{,}819) \Rightarrow \boxed{39{,}3° = \gamma}$$

Calcolo tensione σ_z nel punto + soll. (k, + lontano dall'asse neutro)

k (5,35; 84,65) nel sistema iniziale

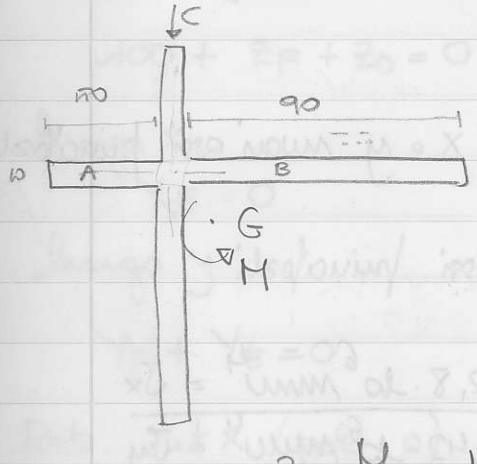
Scrivo le coord. di k nel nuovo sist. di rif. principale

$x = \bar{x}(k)\cdot\cos\alpha + \bar{y}(k)\,\text{sen}\,\alpha = 63{,}64$ mm

$y = -\bar{x}(k)\cdot\text{sen}\,\alpha + \bar{y}(k)\cdot\cos\alpha = 56{,}07$ mm

$$\sigma_z(k) = \frac{3{,}54\cdot\cancel{10^6}}{2{,}8\cdot\cancel{10^6}}\cdot 56{,}07 + \frac{3{,}54\cdot\cancel{10^6}}{3{,}42\cdot\cancel{10^6}}\cdot 63{,}64 = \underline{\underline{136{,}76\ \text{MPa}}}$$

→ EFFETTO MOMENTO TORCENTE



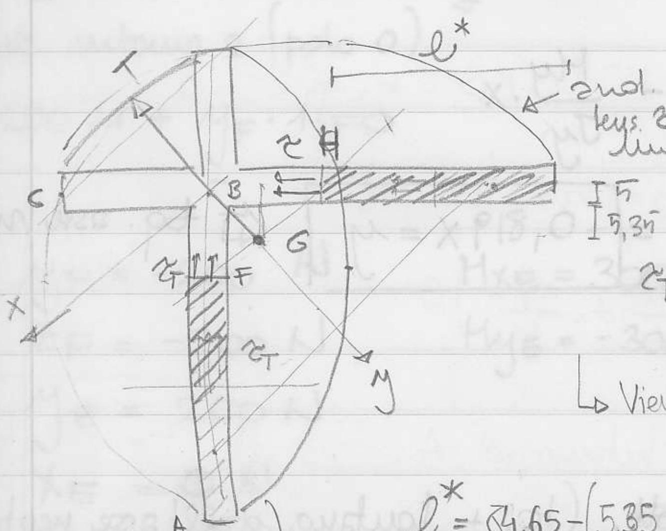
→ le tensioni tangenziali, per una sez. NON TUBOLARE, si determinano come

$$\boxed{\tau_{M_T}^{max} = \frac{3\,M_z\cdot s}{\sum l_i s_i^3}}$$

$$\tau_{max} = \frac{3\cdot M\cdot\cancel{10}}{10^{3\,2}\cdot(50+90+10+50+90)} = \frac{3M}{2{,}9\cdot 10^4} = 1{,}034\cdot 10^{-4}\,M\ \ N/mm^2$$

con M in Nmm

→ EFFETTO DEL TAGLIO (calcolato con Jourawsky) -



$$T_y = -\sqrt{T_{\bar{x}}^2 + T_{\bar{y}}^2} = \underline{\underline{-3\cdot\sqrt{2}\ \text{kN}}}$$

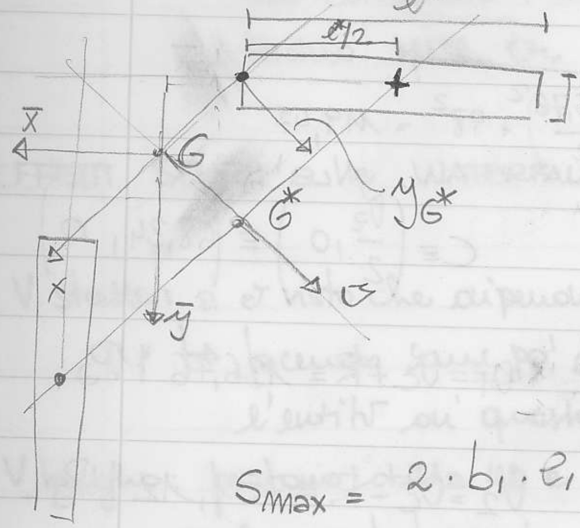
$$\tau_{T(AB)} = \frac{T\cdot S}{2b\cdot J_x}$$

↳ Viene massima nel punto F (o H)

$$l^* = 84{,}65 - (5{,}35 + 5) = 74{,}30\ \text{mm}$$

and. τ lungo ramo verticale -

Determino la pos. del baricentro dei 2 rettangoli (G')



$$\Rightarrow y_G^* = \frac{l^*}{2} \cdot \frac{\sqrt{2}}{2} = 37{,}15 \cdot \frac{\sqrt{2}}{2} = \underline{\underline{26{,}25\ mm}}$$

$$S_{max} = 2 \cdot b_1 \cdot l_1 \cdot y_G^* = 39035\ mm^3$$

$$\tau_{max} = \frac{T \cdot S_{max}}{2b \cdot J_x} = \frac{3 \cdot \sqrt{2} \cdot 10^3 \cdot 39035}{2 \cdot 10 \cdot 2{,}8 \cdot 10^6} = \underline{\underline{2{,}95\ MPa}}$$

↳ Le tensioni da taglio vengono quindi trascurabili. il punto + caricato rimane quindi il punto k

$\sigma_z(k) = 136{,}76\ MPa$

$\tau_{Mt}(k) = 1{,}034 \cdot 10^{-1}\ M\ \ (MPa)$ ← ipotesi che sia coincidente con k il punto + sollecitato.

↳ Taglio nullo

$$\sigma_{eq} = \sqrt{\sigma_z^2 + 4\,\tau_{MAX}^2} = \frac{\sigma_{amm}}{C_s.}$$

$$\left(\sigma_z^2 + 4\,\tau_{MAX}^2\right) \cdot 1{,}25^2 = 300^2$$

$$4\,\tau_{max}^2 = \left(\frac{300}{1{,}25}\right)^2 - \sigma_{z\,max}^2 =$$

$$\tau_{max} = \frac{1}{2} \cdot \sqrt{\left(\frac{300}{1{,}25}\right)^2 - \sigma_{z\,max}^2} = 98{,}61\ MPa$$

$$M = \frac{\tau_{max}}{1{,}034 \cdot 10^{-1}} = \underline{953\ Nm}$$

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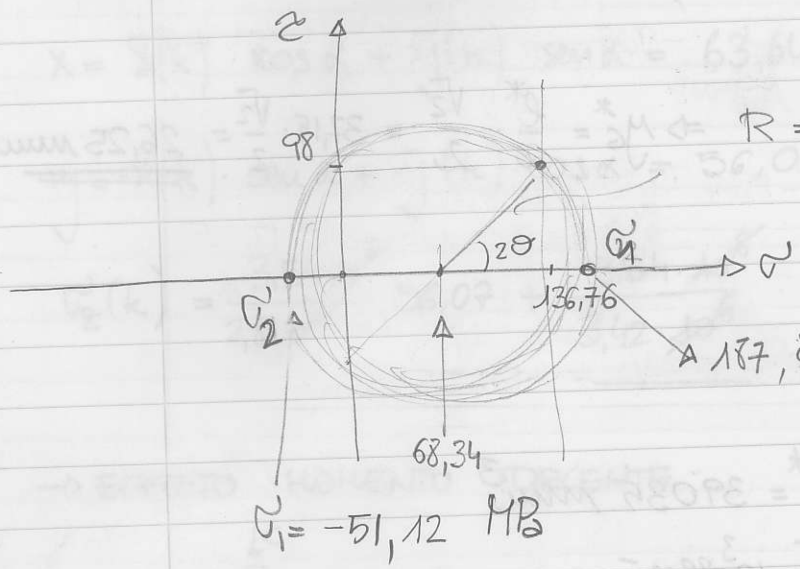
CERCHI DI MOHR:



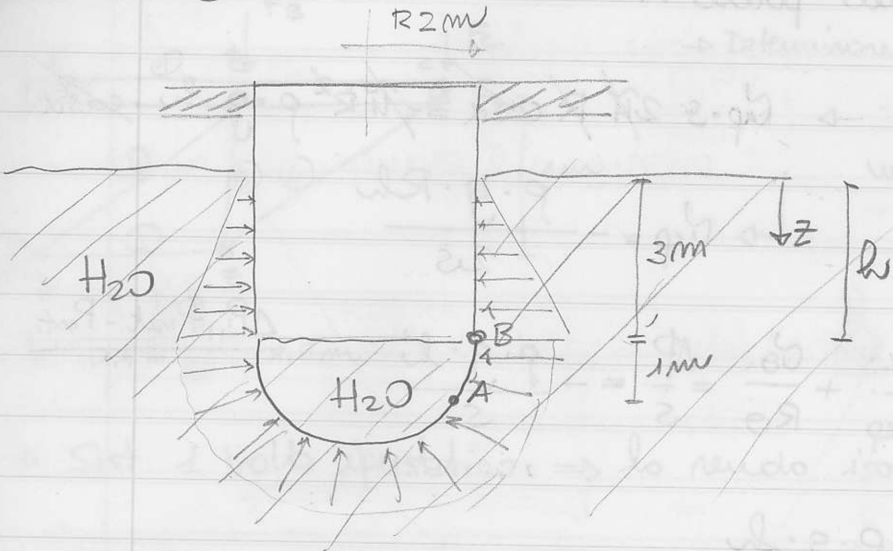
$$R = \sqrt{\left(\frac{136,76}{2}\right)^2 + 98^2} = 119,47$$

$$C \equiv \left(\frac{\sigma_z}{2}, 0\right) \equiv (68,34,\ 0)$$

$$\sigma_1 = \sigma_C + R = 136,76 \text{ MPa}$$

$$\sigma_2 = \sigma_C - R = -51,12 \text{ MPa}$$

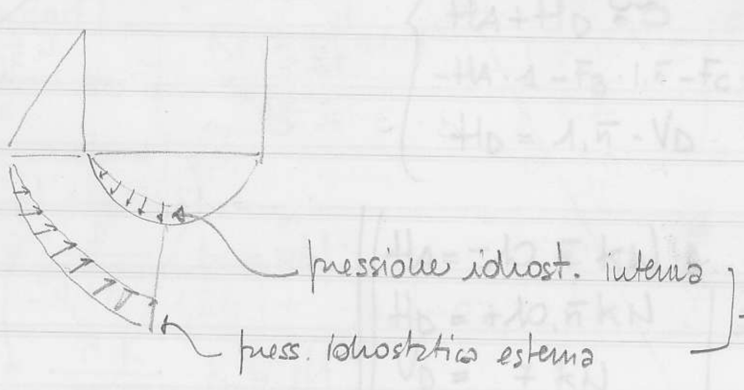
ESERCIZIO 4



→ Press. idrostatica in B : $p = \rho \cdot g \cdot z_B = 1000 \cdot 9{,}81 \cdot 3 = 29430$ Pa

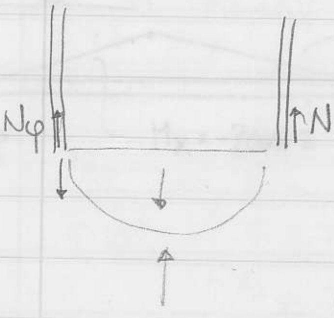
kg/m³

→ Nel tratto sferico la pressione RIMANE COSTANTE, perché la quota di pressione che aumenta all'esterno (x via della profondità) è equilibrata dall'acqua che si trova all'interno della calotta sferica



→ Tutta la calotta sferica è soggetta a pressione costante pari a $p_B = 29430$

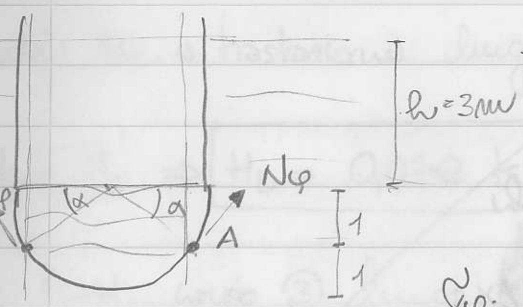
→ Calcolo tens. membranali in B :



$$-2\cancel{\pi} R \cdot s \cdot \sigma_\varphi = -h\, \rho \cdot \cancel{\pi} R^{\cancel{2}} g \Rightarrow \sigma_\varphi = \frac{-\rho g R h}{2s} = -9{,}81 \text{ MPa}$$

$$\cancel{\frac{\sigma_\varphi}{R_\varphi}} + \frac{\sigma_\theta}{R_\theta} = -\frac{p}{s} \Rightarrow \sigma_\theta = \frac{-\rho g R_\theta h}{s} = -19{,}62 \text{ MPa}$$

→ Calcolo ora le tensioni nel punto A



$$\rightarrow \sigma_\varphi \cdot s \, 2\cancel{\pi} R \cancel{\cos^2\alpha} = -\cancel{\pi} R^2 \cdot \rho \cdot g \cdot h \cdot \cancel{\cos^2\alpha}$$

$$\hookrightarrow \sigma_\varphi = -\frac{\rho \cdot g \cdot R h}{2s}$$

$$\frac{\sigma_\varphi}{R_\varphi} + \frac{\sigma_\theta}{R_\theta} = \frac{\Delta p}{s} = -\frac{\rho \cdot g \cdot h}{s}$$

ΔP: $P_{int} - P_{ext}$

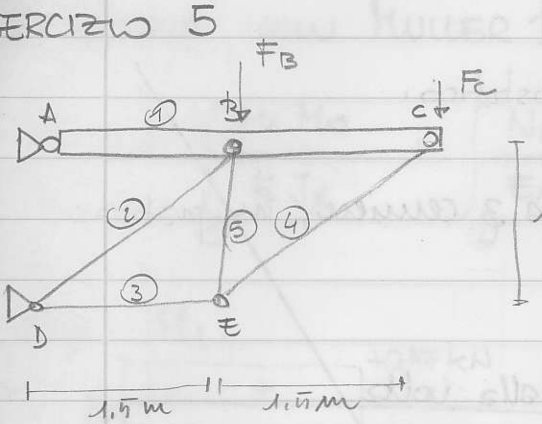
$$\Rightarrow -\frac{\rho \cdot g h}{2s} + \frac{\sigma_\theta}{R} = -\frac{\rho \cdot g \cdot h}{s}$$

$$\Rightarrow \sigma_\theta = \frac{R}{s}\left(-h\rho \cdot g + \frac{\rho g h}{2}\right) = \frac{-\rho \cdot g \cdot R \cdot h}{2s}$$

$$\Rightarrow \sigma_{\theta_A} = \frac{-\rho \cdot g \cdot R \cdot h}{2s} = \frac{-1000 \cdot 9{,}81 \cdot \cancel{2} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{3} \cdot 10^{-3}} = -9{,}81 \cdot 10^6 \, Pa = \underline{-9{,}81 \, MPa}$$

$$\sigma_{\varphi(A)} = \frac{-\rho \cdot g \cdot R \, h}{2s} = \sigma_\theta = \underline{-9{,}81 \, MPa}$$

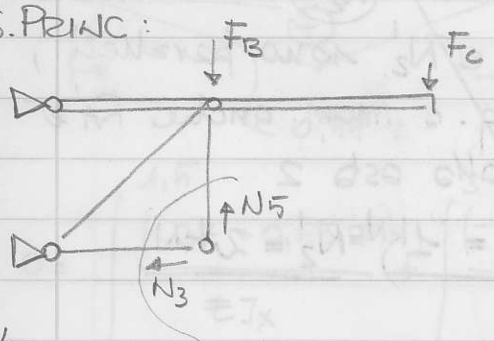
ESERCIZIO 5



→ Determinare la forza in $\overline{CE}$

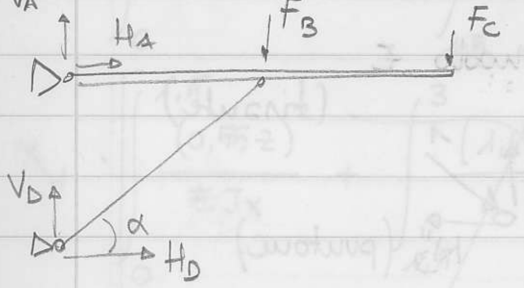
→ Sist. 1 volta iperstatico ⇒ lo rendo isostatico sconnettendo l'asta $\overline{EC}$

S. PRINC:



Eq. del nodo E : $\vec{N_3} + \vec{N_5} = \vec{0}$
vero solo se $\vec{N_3} = \vec{0}$; $\vec{N_5} = \vec{0}$

le aste 3 e 5 sono scariche

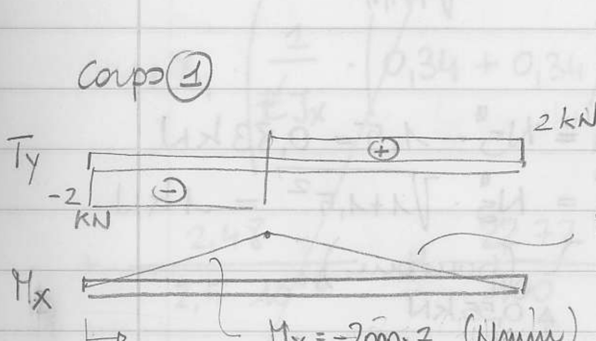


$$\begin{cases} V_A + V_D = F_B + F_C \\ H_A + H_D = 0 \\ -H_A \cdot 1 - F_B \cdot 1,5 - F_C \cdot 3 = 0 \\ H_D = 1,5 \cdot V_D \end{cases}$$

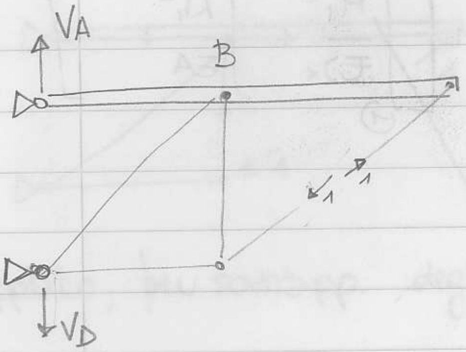
$$\begin{cases} H_A = -10,5 \text{ kN} \\ H_D = +10,5 \text{ kN} \\ V_D = 7 \text{ kN} \\ V_A = -2 \text{ kN} \end{cases}$$

$\Rightarrow N_2 = \sqrt{7^2 + 10,5^2} = \underline{12,62 \text{ kN}}$
puntone

Corpo ①



5-2 SISTEMA AUSILIARIO

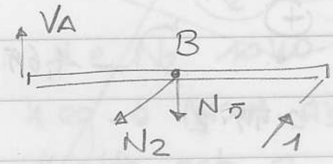


Globalmente il sistema è soggetto ad un sistema di forze con risultante nulla (anche per i momenti)

↳ Le reazioni vincolari quindi dovranno essere un sist. di forze diretto in diret. verticale e contrapposte.

→ Eq. ① (a mom. rispetto B)

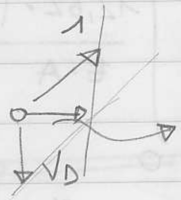
$$V_A \cdot \cancel{1,5} = 1 \cdot \cancel{1,5} \cdot \frac{1}{\sqrt{1^2+1,5^2}} = 0,55\ N$$



$V_D = 0,55$

$\underline{N_2 = 1\ N}$ (da eq. a traslaz. in dir. oriz.)

→ Nodo D



$N_3 = -0,83\ N$

$N_5 = 0,55\ N$

RICAPITOLANDO

ASTA	S. PRINC.	S. AUS
2	12'620 N (P)	1 N (T)
3	SCARICA	0,83 N (P)
4	"	0,55 N (T)
5	"	1 N (P)

$$\delta_{1P} = 2 \cdot \int_0^{1500} \frac{-2000 \cdot 0{,}55 \cdot z^2}{EJ_x} dz + \int_0^{1803} \frac{-12'620 \cdot 1 \, dz}{EA} = -4{,}95 - 1{,}14 = \underline{-6{,}09 \text{ mm}}$$

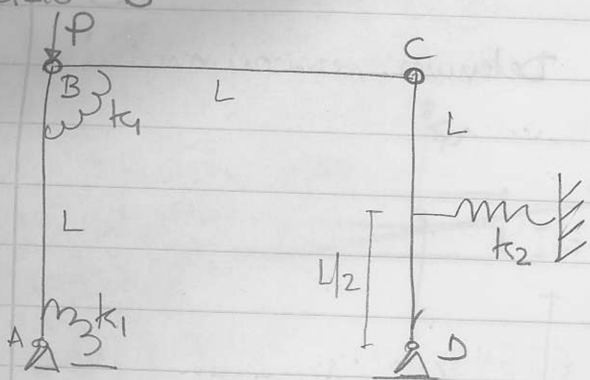
$$\delta_{11} = 2 \cdot \int_0^{1500} \frac{0{,}55^2 z^2 dz}{EJ_x} + 2 \cdot \frac{1 \cdot 1803}{EA} + \frac{0{,}55^2 \cdot 1000}{E \cdot A} + \frac{0{,}83^2 \cdot 1500}{EA} =$$

$$= 0{,}00136 + 0{,}00025 = 0{,}0016 \text{ mm}$$

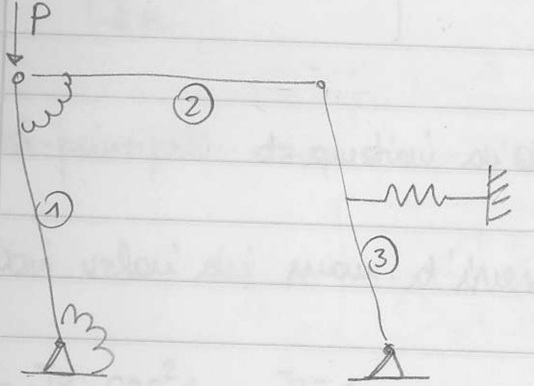
$$X_1 = -\frac{\delta_{1P}}{\delta_{11}} = + \frac{6{,}09}{0{,}0016} = +3790$$

$$\rightarrow \boxed{N_4 = +3790 \text{ N}}$$

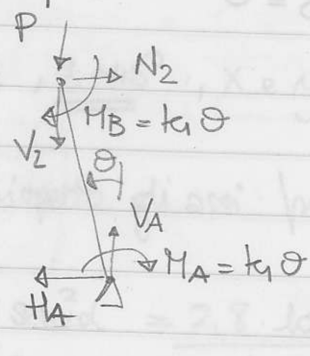
ESERCIZIO 6



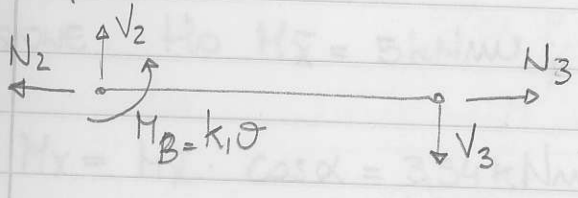
↳ Considero corpi rigidi e una conf. deformata di un angolo θ piccolo



Eq. corpo 1

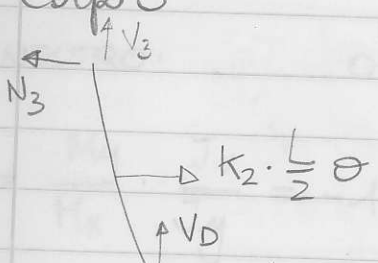


Corpo 2



$$\begin{cases} V_2 = V_3 \\ V_3 = \frac{M}{L} = \frac{k_1\theta}{L} \\ N_2 = N_3 \end{cases}$$

Corpo 3



$$N_3 \cdot \cancel{L} - V_3 \cdot \cancel{L}\theta - k_2 \frac{L^2}{4}\theta = 0$$

$$\Rightarrow N_3 = \left(V_3 + \frac{k_2 L}{4}\right)\theta = \left(\frac{k_1\theta}{L} + \frac{k_2 L}{4}\right)\theta$$

↳ Ritorno all'eq. corpo 1 a rot. intorno ad A

$$\boxed{P = \frac{9}{4}k_2 L}$$

$$P \cdot L \cdot \theta + V_2 L \cdot \theta - 2k_1\theta - \frac{k_1\theta^2}{\cancel{L}}L - \frac{k_2\theta L^2}{4} = 0$$

$$P \cdot L\cancel{\theta} + \cancel{k_1\theta^2} - 2k_1\cancel{\theta} - \cancel{k_1\theta^2} - \frac{k_2\cancel{\theta}L^2}{4} = 0 \Rightarrow P \cdot \cancel{L} = 2k_1 + \frac{k_2 L^2}{4} = \frac{9}{4}k_2 L^{\cancel{2}}$$