



**CORSO DI
PROGETTAZIONE ASSISTITA DELLE STRUTTURE
MECCANICHE**

**PARTE III A
REV.: 03 del 15 marzo 2012**

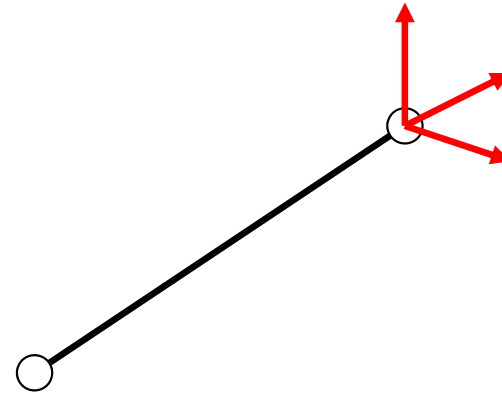
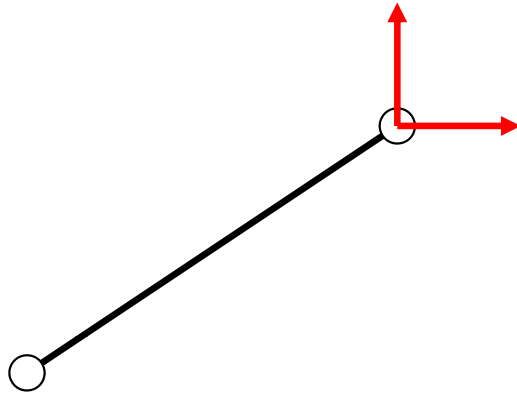
**PRINCIPALI TIPI DI ELEMENTO E LORO
IMPIEGO (PARTE A)**

PRINCIPALI TIPI DI ELEMENTO

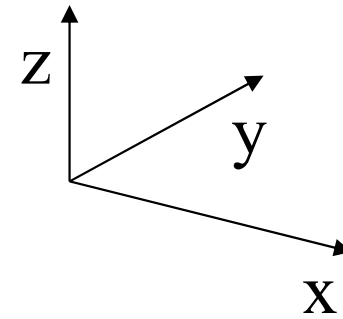
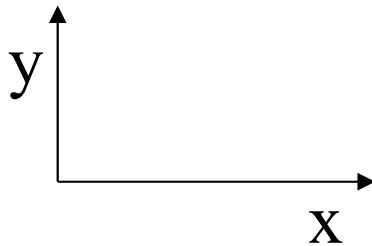
2D

3D

ASTA



Travature reticolari

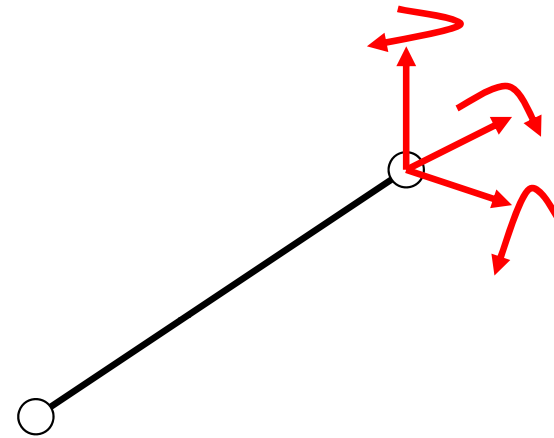
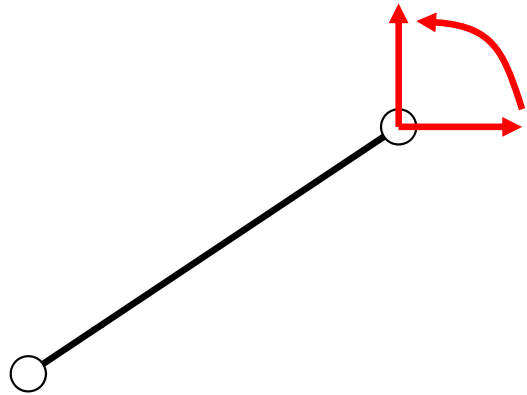


PRINCIPALI TIPI DI ELEMENTO

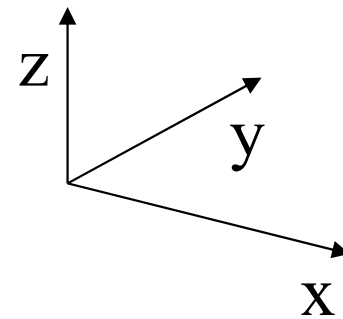
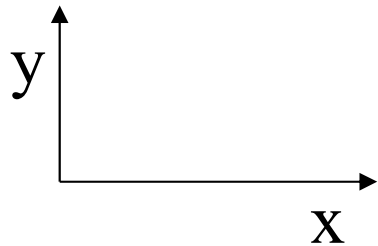
2D

3D

TRAVE



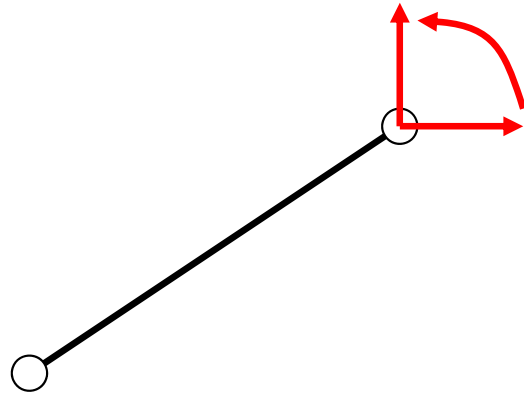
Telai



PRINCIPALI TIPI DI ELEMENTO

2D

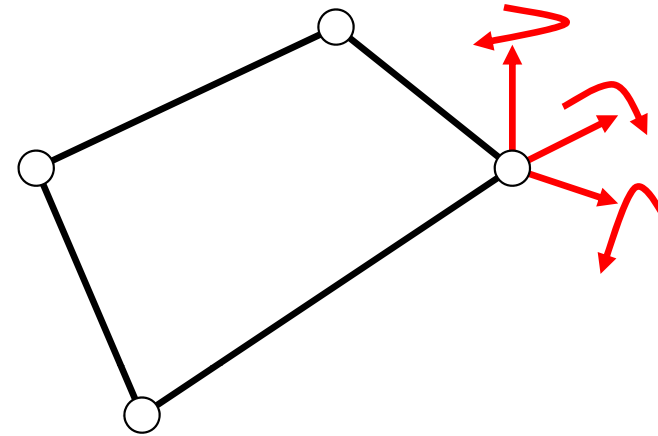
GUSCIO



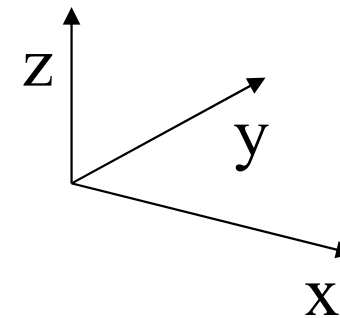
Piastra/guscio assialsimmetrico



3D



Piastra/guscio 3D

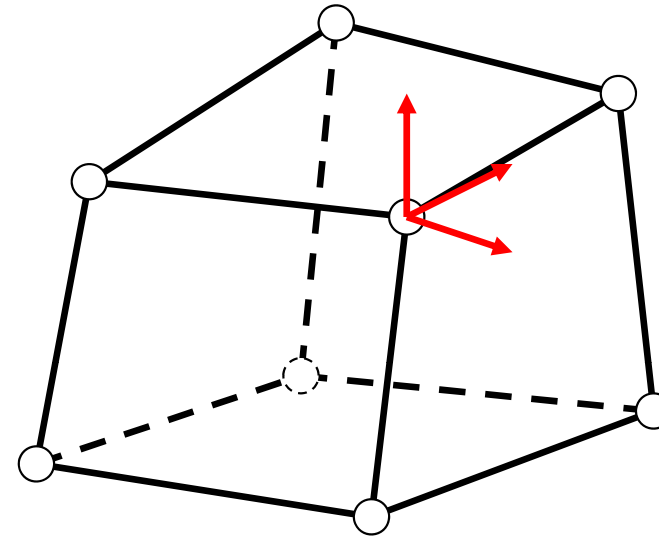
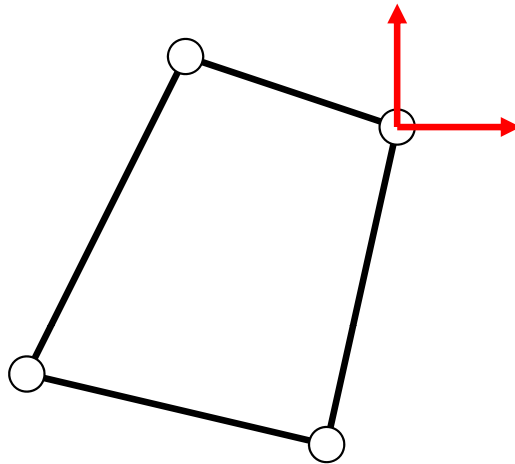


PRINCIPALI TIPI DI ELEMENTO

2D

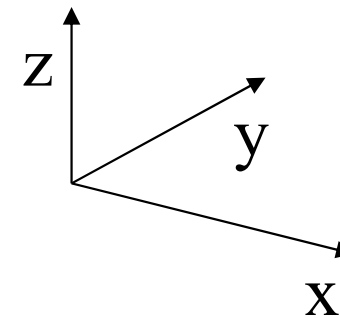
3D

SOLIDO



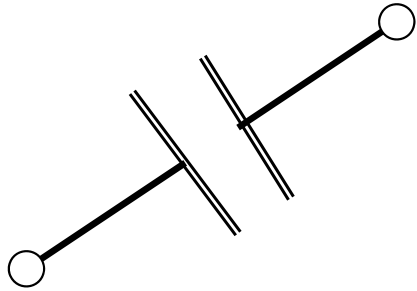
Pb. di Elasticità piana

Pb. di Elasticità 3D



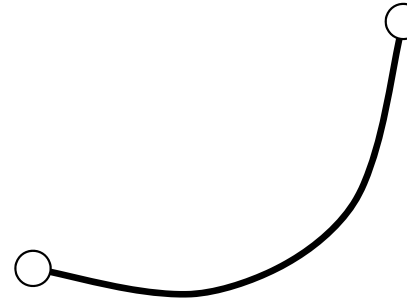
ALTRI TIPI COMUNI DI ELEMENTO

“GAP”



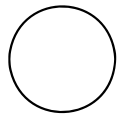
Pb. contatto

“PIPE”



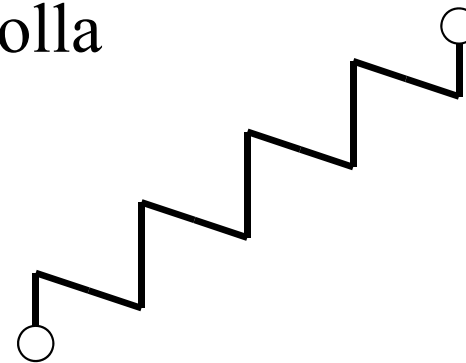
Tubazioni

Massa



Masse concentrate

Molla



Elementi elastici

ELEMENTO ASTA/1

Figure 1.1 LINK1 Geometry

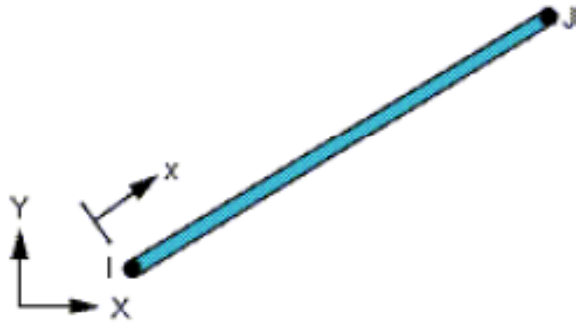
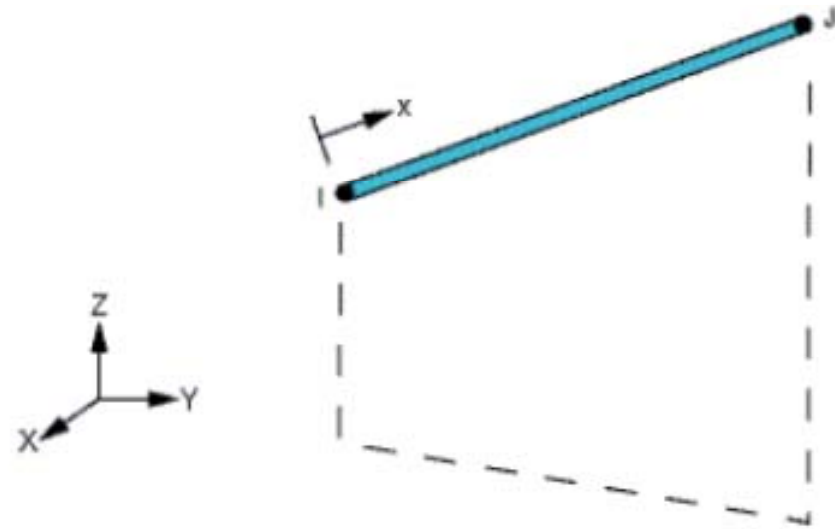


Figure 8.1 LINK8 Geometry



Travature reticolari piane e spaziali

- *sola forza normale*
- *2 nodi*
- *2 o 3 g.d.l /nodo*
- *carichi applicabili solo nei nodi*
- *caratteristiche geometriche richieste: A*

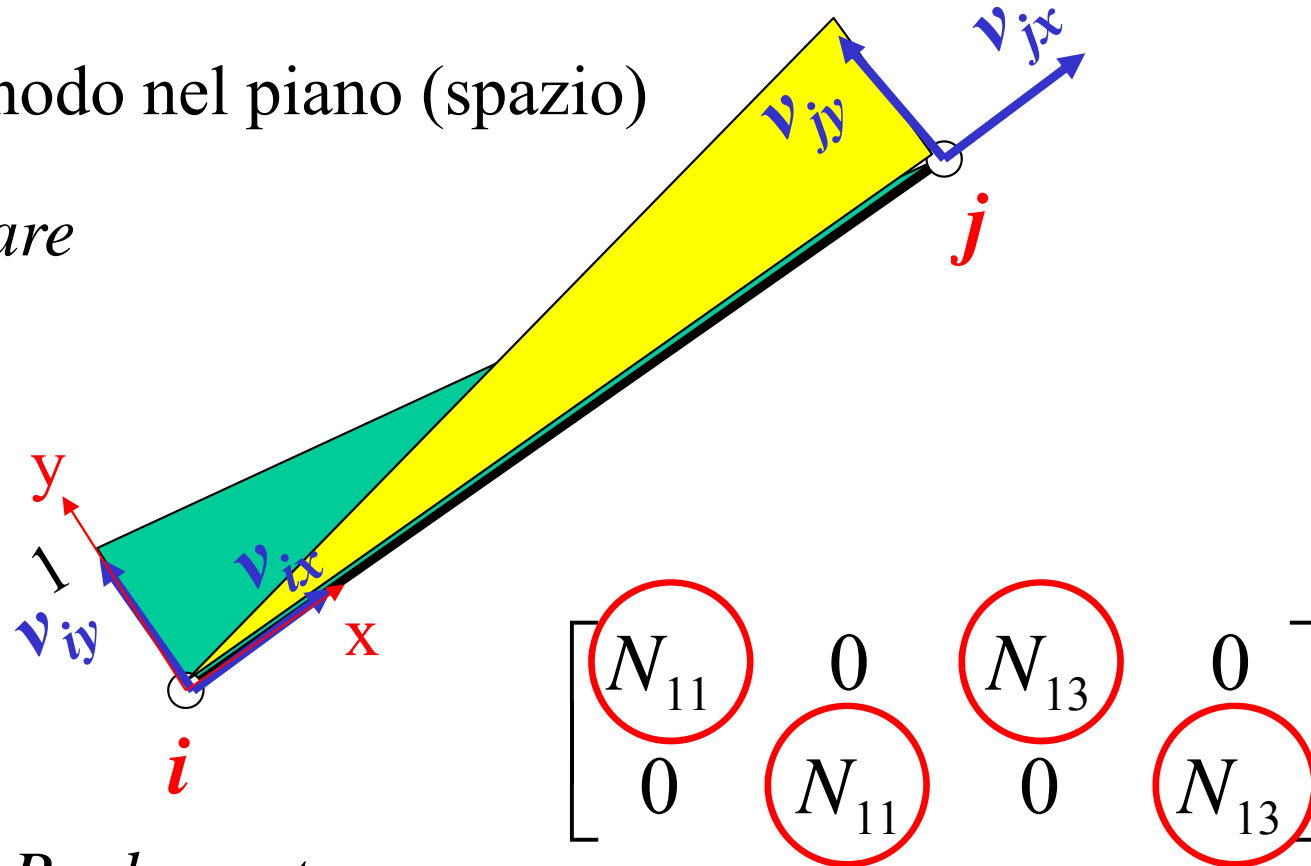
ELEMENTO ASTA/2

N° 2 nodi

N° 2(3) g.d.l. per nodo nel piano (spazio)

F.ne di forma lineare

$$N_{11} = A_{11} + B_{11}x$$



Espressione nel S.R. elemento:

$$N_{11} = (L-x)/L$$

$$N_{13} = x/L$$



ELEMENTO ASTA/3

OSSERVAZIONE: La soluzione ottenuta è **esatta**, nel senso che rappresenta senza errori lo stato di tensione/deformazione di un membro di una travatura reticolare.

Elemento asta

$$v_x(x) = v_{ix} \cdot \frac{(L-x)}{L} + v_{jx} \cdot \frac{x}{L}$$

$$\varepsilon = \frac{\partial}{\partial x} \left(v_{ix} \cdot \frac{(L-x)}{L} + v_{jx} \cdot \frac{x}{L} \right) = \frac{v_{jx} - v_{ix}}{L} = \text{cost}$$

$$\varepsilon = \frac{N}{EA} = \text{cost}$$

$$v_{jx} = v_{ix} + \varepsilon \cdot L$$

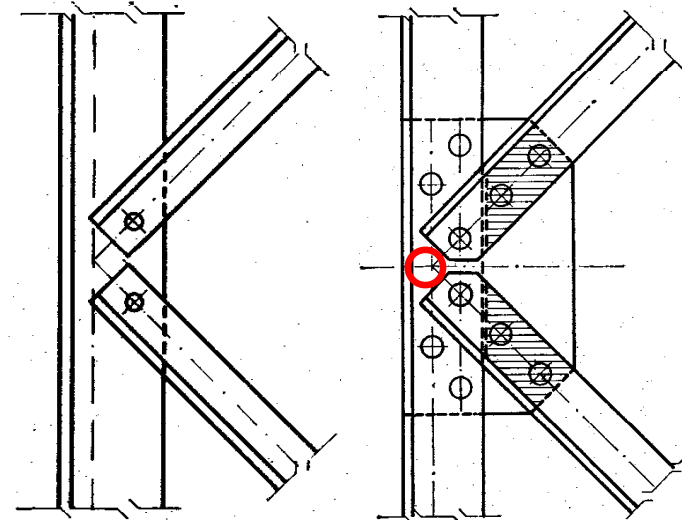
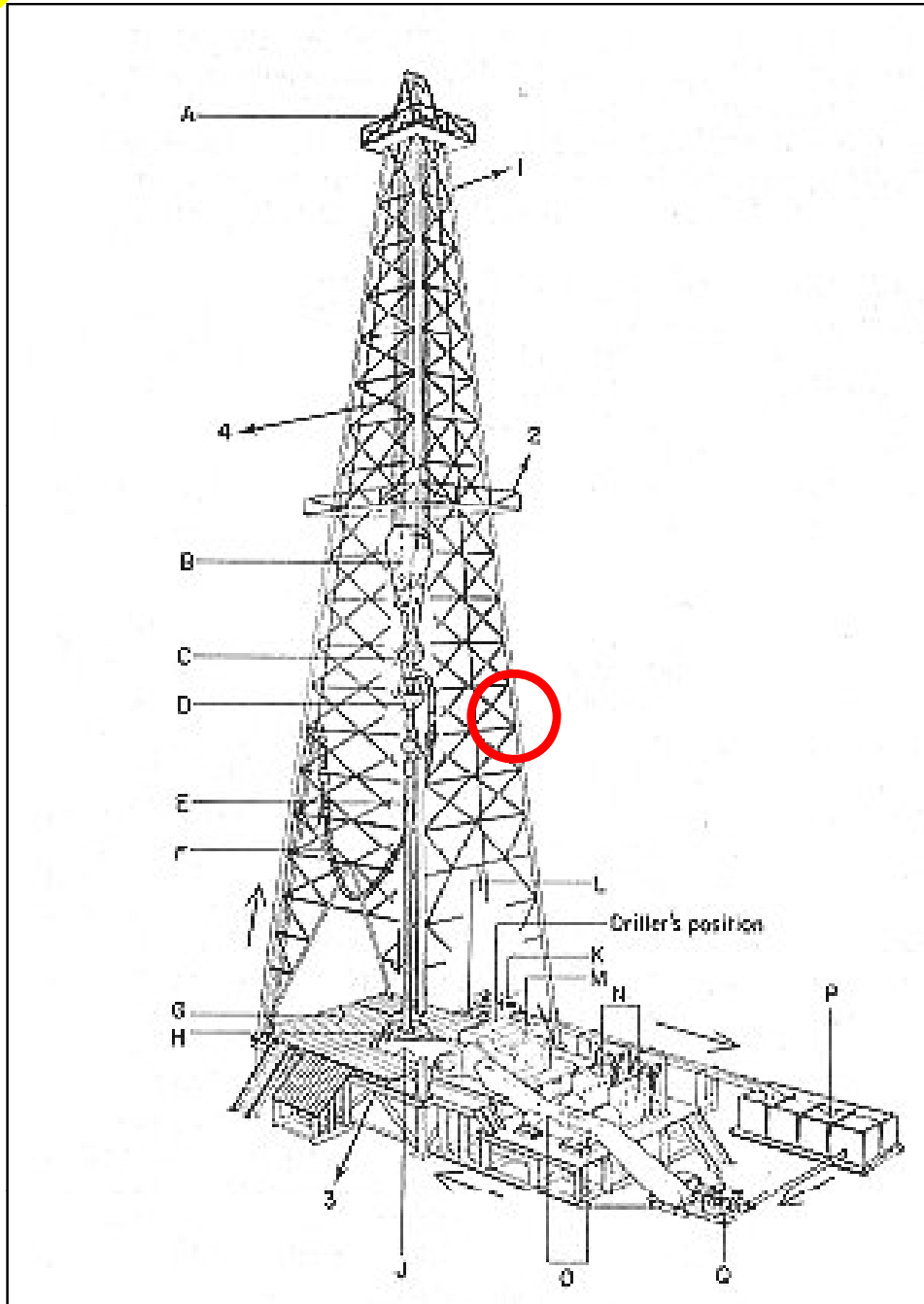
$$\varepsilon = \frac{v_{jx} - v_{ix}}{L}$$

Membro travatura reticolare

ELEMENTO ASTA/4 - TRALICCIO

Traliccio di sostegno per batterie di perforazione petrolifera.

Questo tipo di strutture viene tradizionalmente trattato con modelli a travatura reticolare, assimilando i “nodi” a cerniere.

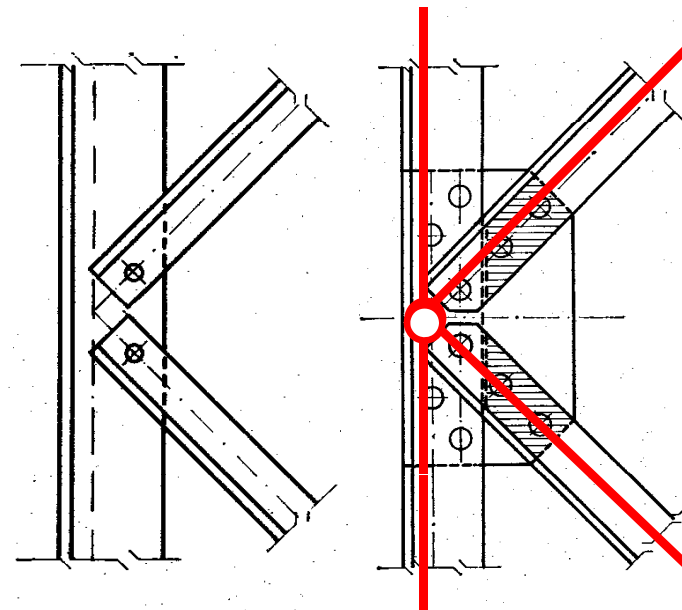
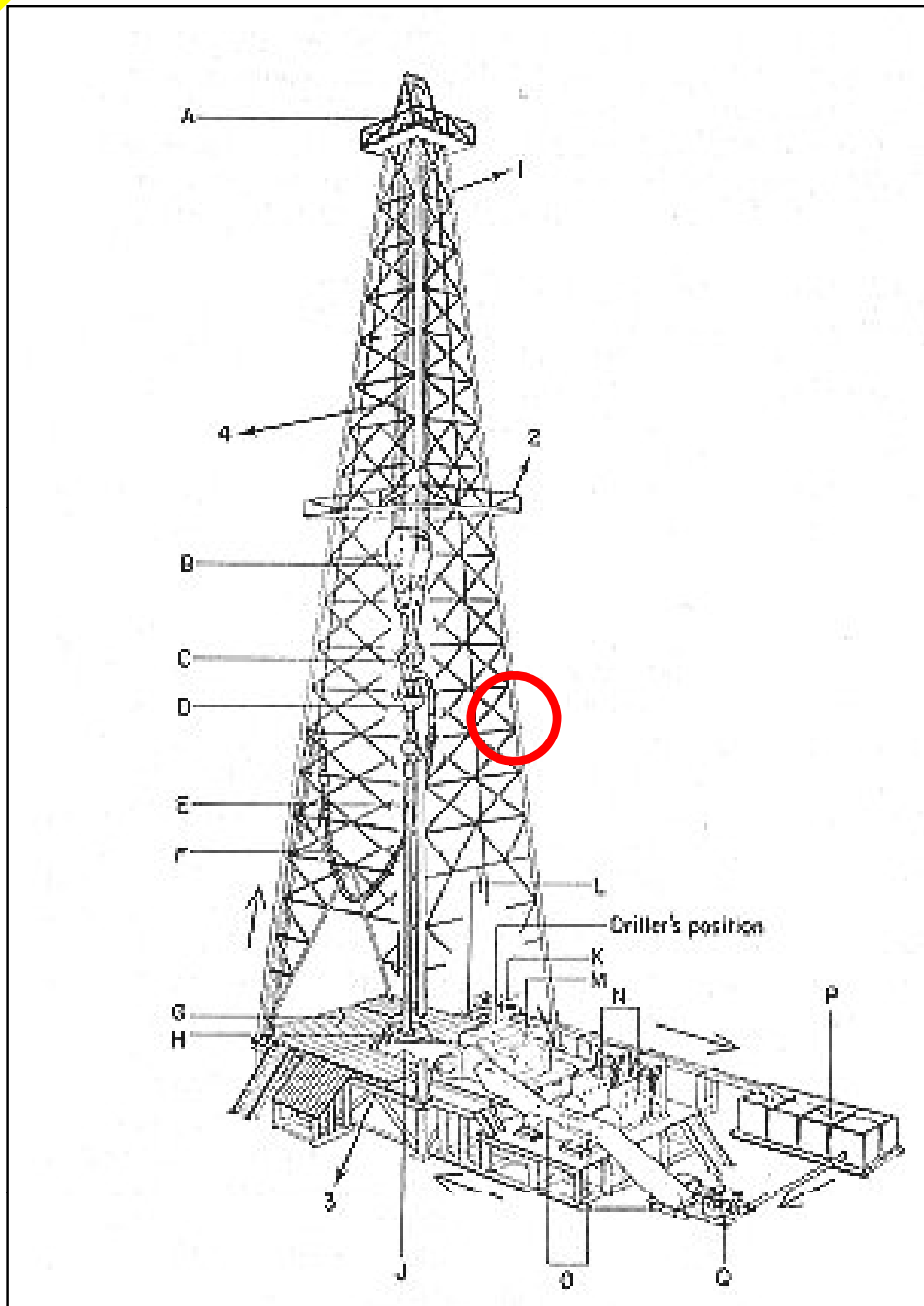


ELEMENTO ASTA/4 - TRALICCIO

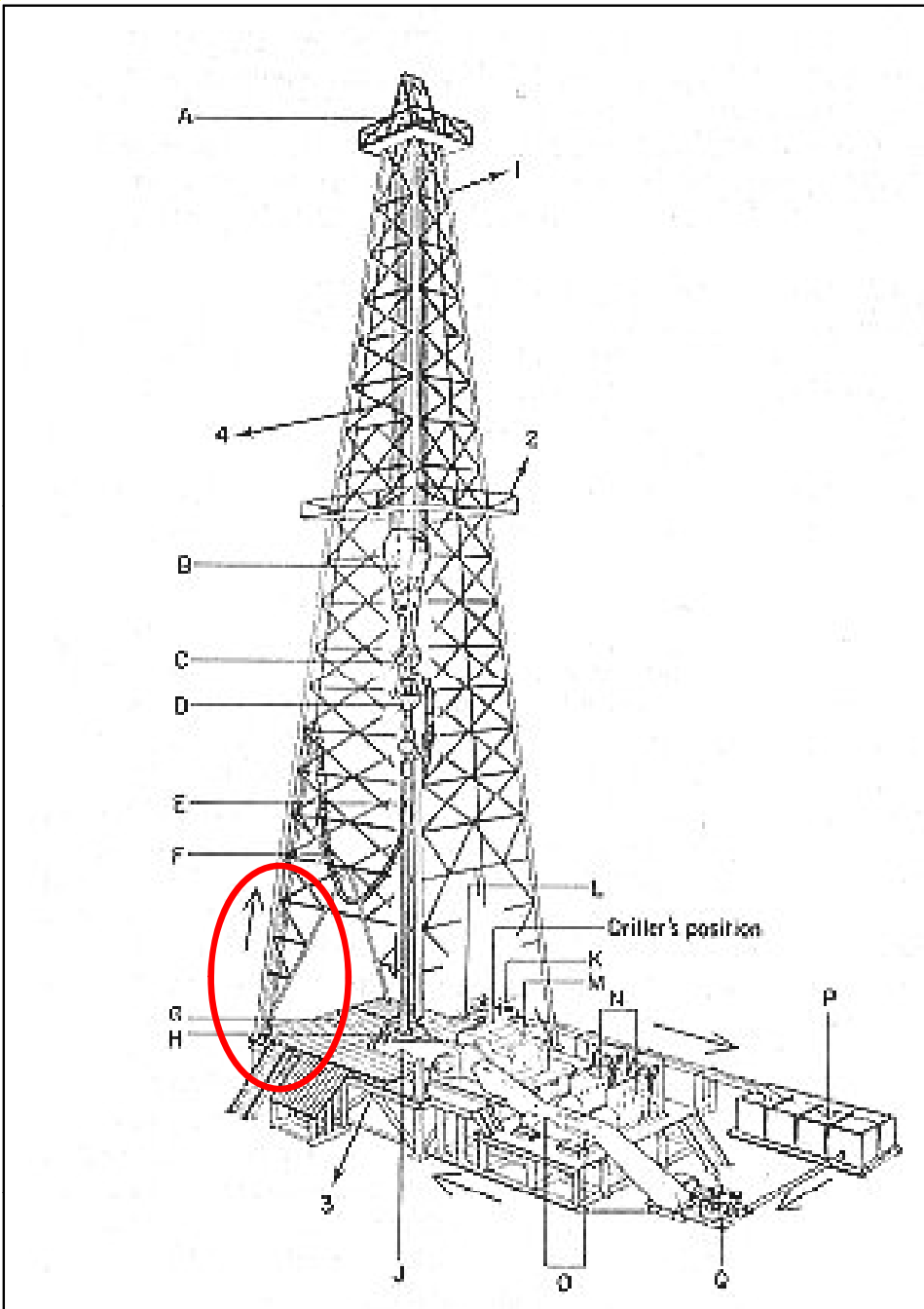
Traliccio di sostegno per batterie di perforazione petrolifera.

Il modello è giustificabile con:

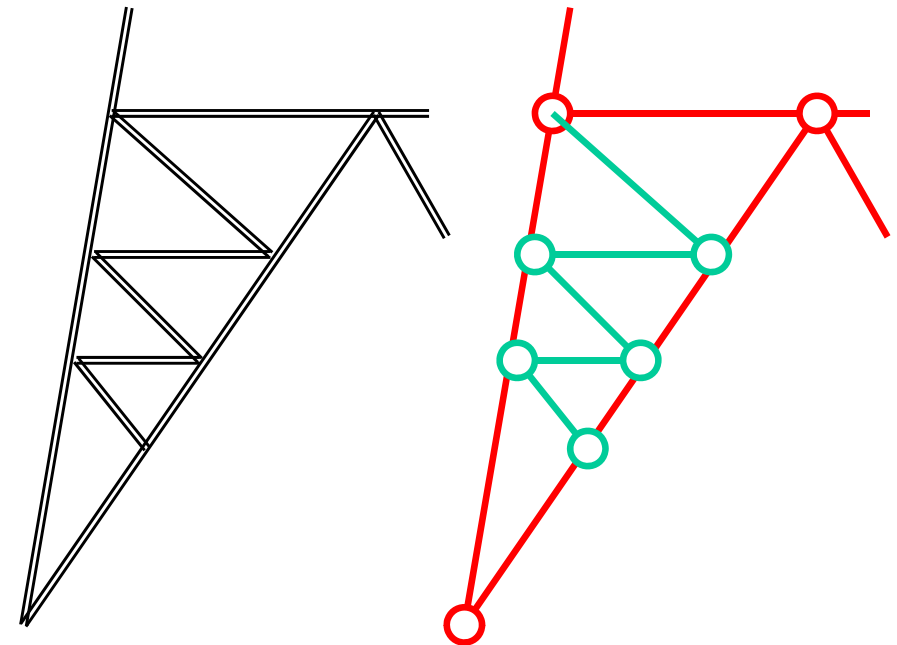
- bassa rigidezza flessionale delle aste
- giochi tra bulloni e fori



ELEMENTO ASTA/5 - TRALICCIO



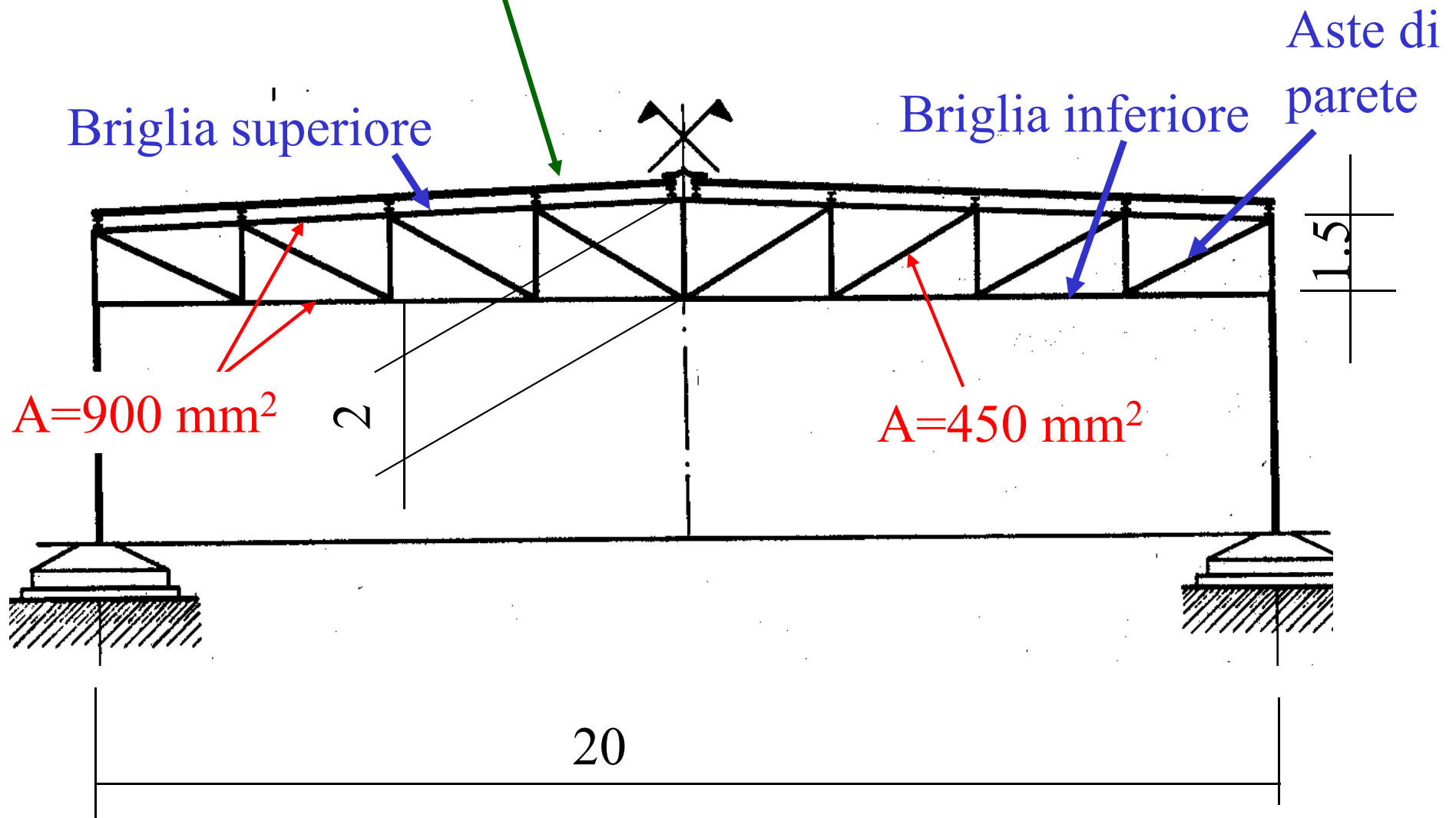
Nel fare il modello si escludono solitamente le aste che non hanno una funzione strutturale (rompitratta)



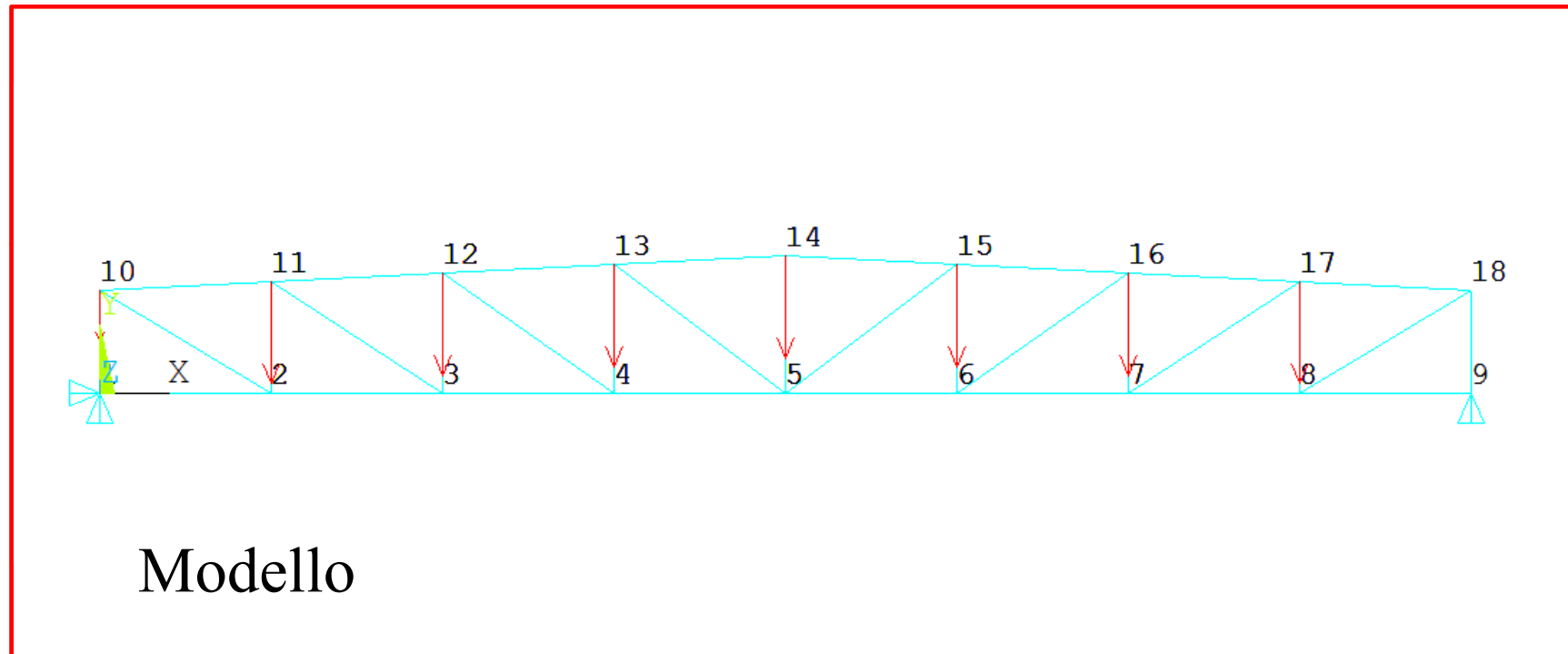
Modello di calcolo

ELEMENTO ASTA/6 – ALTRE STRUTTURE

Peso copertura = 10 KN/m



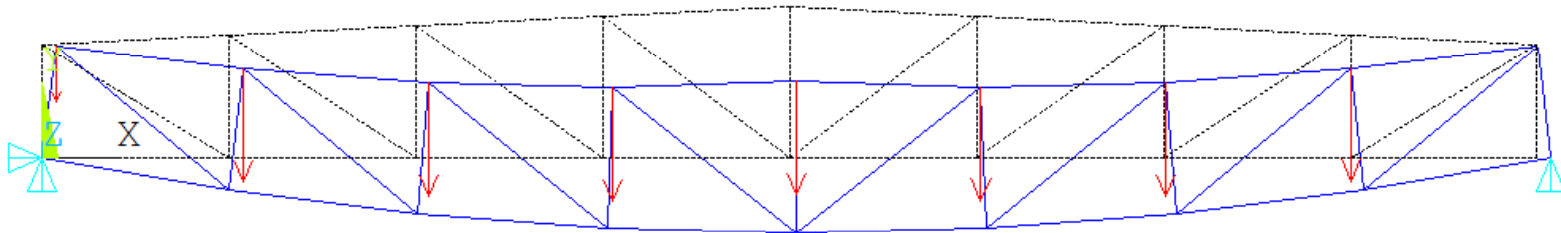
ELEMENTO ASTA/7 – ALTRE STRUTTURE



File di comandi: capriata_reticolare_piana.txt

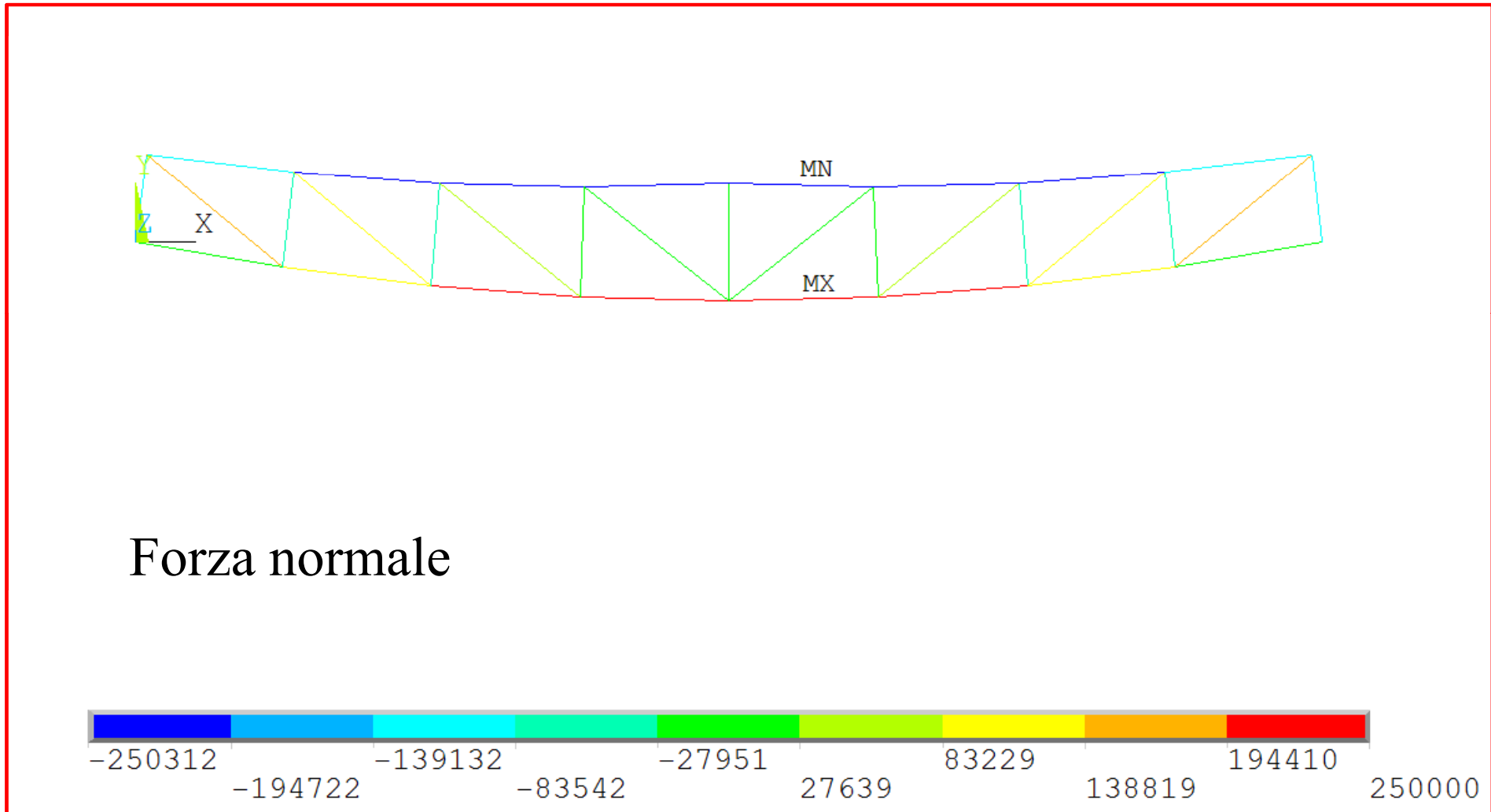
ELEMENTO ASTA/7 – ALTRE STRUTTURE

Deformata



File di comandi: capriata_reticolare_piana.txt

ELEMENTO ASTA/7 – ALTRE STRUTTURE



File di comandi: capriata_reticolare_piana.txt



ELEMENTO ASTA/8

Dati di input per l'elemento asta 3D (8) di ANSYS

Nodes

I, J

Degrees of Freedom

UX, UY, UZ

Real Constants

AREA - Cross-sectional area

ISTRN - Initial strain

Material Properties

EX, ALPX (or CTEX or THSX), DENS, DAMP

ELEMENTO ASTA/9

Accesso ai risultati per l'elemento asta 3D – Comando ETABLE

Table 8.2 LINK8 Item and Sequence Numbers

Output Quantity Name	ETABLE and ESOL Command Input			
	Item	E	I	J
SAXL	LS	1	-	-
EPELAXL	LEPEL	1	-	-
EPHAXL	LEPTH	1	-	-
EPSWAXL	LEPTH	2	-	-
EPINAXL	LEPTH	3	-	-
EPPLAXL	LEPPL	1	-	-
EPCRAXL	LEPCR	1	-	-
SEPL	NLIN	1	-	-
SRAT	NLIN	2	-	-
HPRES	NLIN	3	-	-
EPEQ	NLIN	4	-	-
MFORX	SMISC	1	-	-
FLUEN	NMISC	-	1	2
TEMP	LBFE	-	1	2

ETABLE,N,SMISC,1

! estrae la "forza normale" dal data base

ETABLE, Lab, Item, Comp



ELEMENTO ASTA/10

Accesso ai risultati per l'elemento asta 3D – Comando ETABLE

Table 8.2 LINK8 Item and Sequence Numbers

Output Quantity Name	ETABLE and ESOL Command Input			
	Item	E	I	J
SAXL	LS	1	-	-
EPELAXL	LEPEL	1	-	-
EPHAXL	LEPTH	1	-	-
EPSWAXL	LEPTH	2	-	-
EPINAXL	LEPTH	3	-	-
EPPLAXL	LEPPL	1	-	-
EPCRAXL	LEPCR	1	-	-
SEPL	NLIN	1	-	-
SRAT	NLIN	2	-	-
HPRES	NLIN	3	-	-
EPEQ	NLIN	4	-	-
MFORX	SMISC	1	-	-
FLUEN	NMISC	-	1	2
TEMP	LBFE	-	1	2

ETABLE,SN,LS,1

! estrae il dato "tensione assiale" dal data base

ETABLE, Lab, Item, Comp

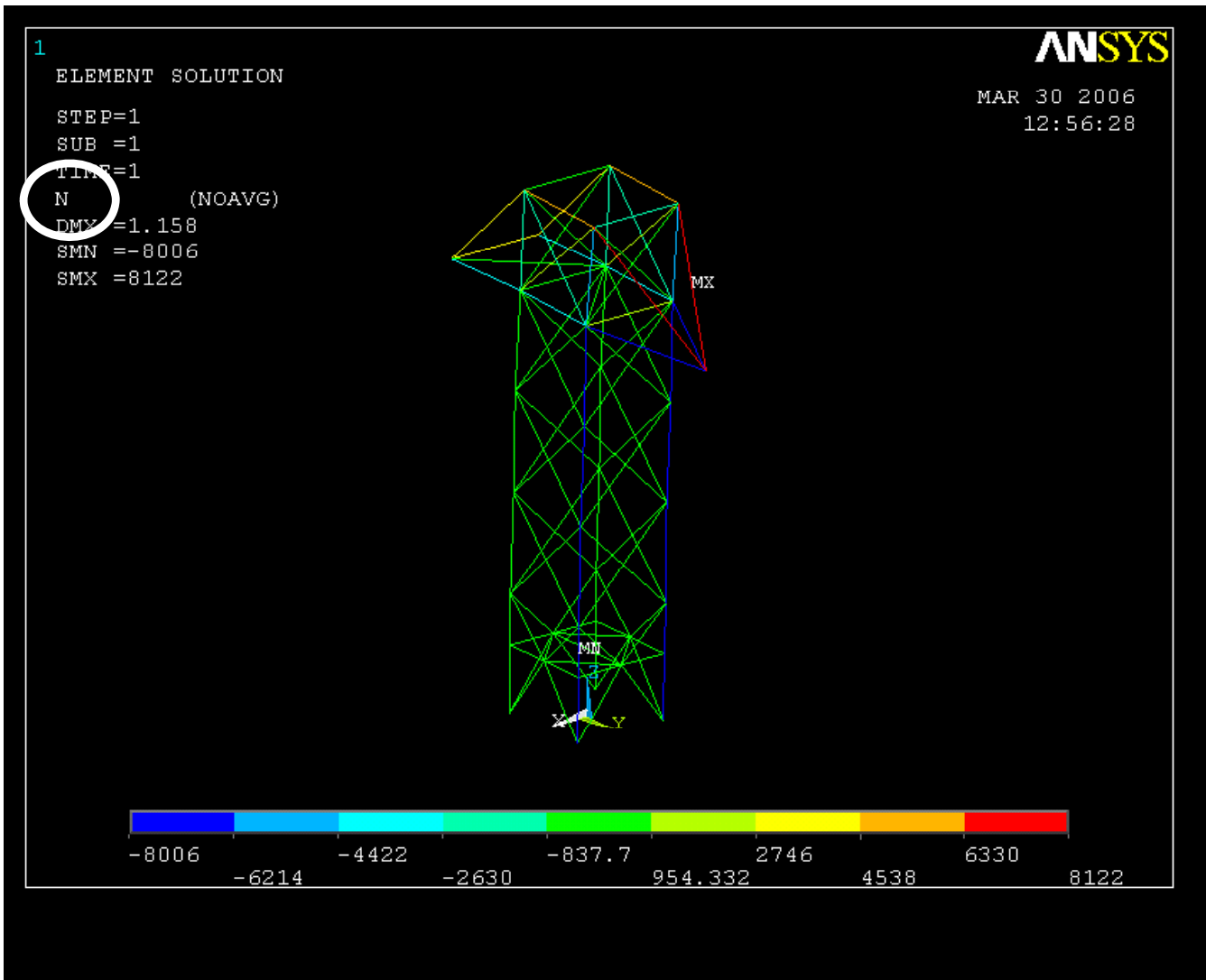
ELEMENTO ASTA/11

Rappresentazione grafica risultati – Comando PLETAB

PLETAB, *Itlab*, *Avglab*

ETABLE,N,SMISC,1

PLETAB,N



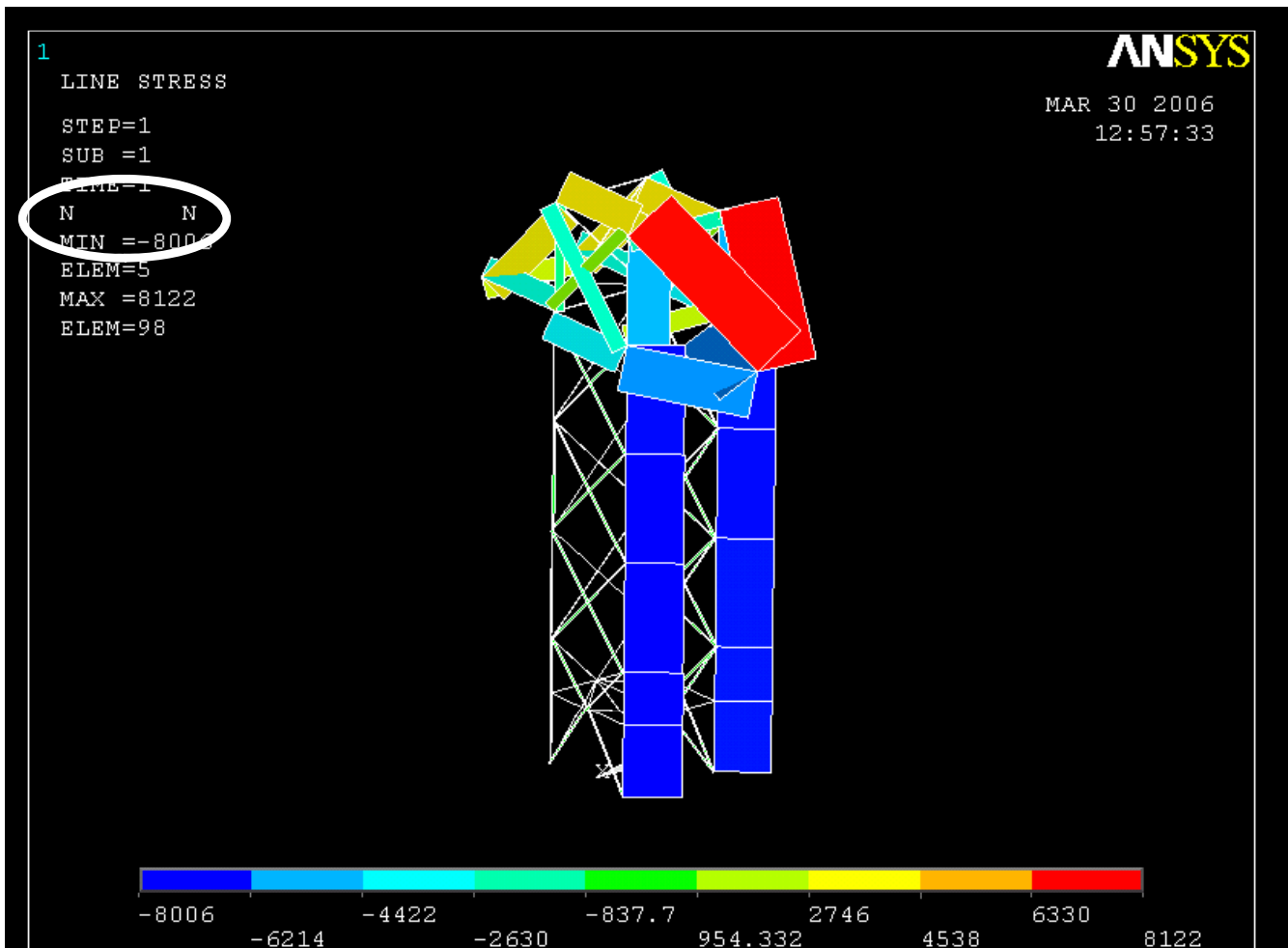
ELEMENTO ASTA/12

Rappresentazione grafica risultati – Comando PLLS

PLLS, LabI, LabJ, Fact, KUND

ETABLE, N, SMISC, 1

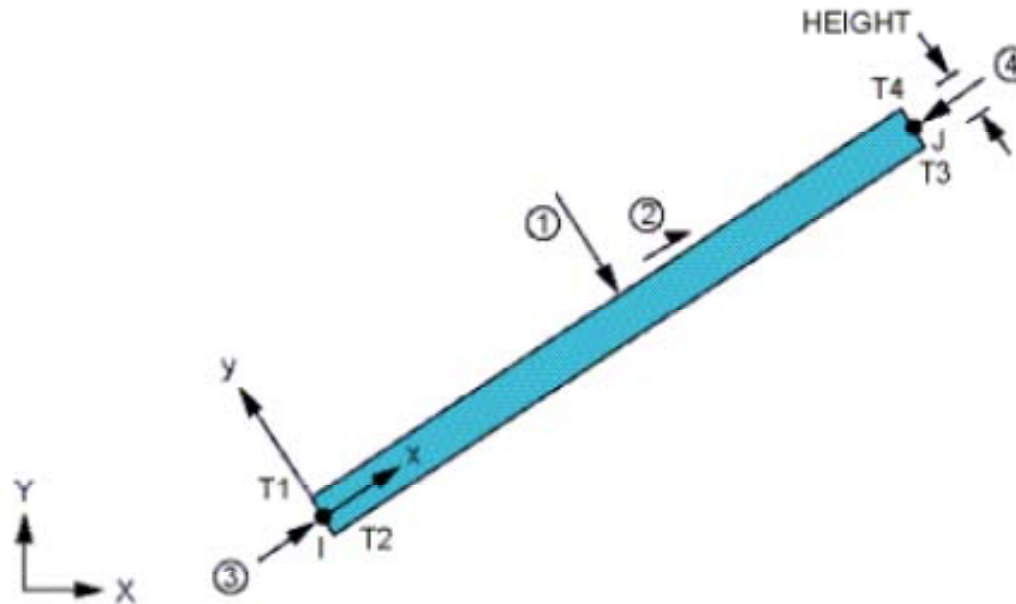
PLLS, N, N



ELEMENTO TRAVE/1

Figure 3.1 BEAM3 Geometry

2D



Telai piani

- 2 nodi
- 3 g.d.l /nodo
- carichi concentrati e distribuiti
- Car. geometriche: A , J_x , ...

Il piano x,y deve contenere:

- fibre baricentriche travi
- rette di azione dei carichi
- uno degli assi principali di inerzia delle sezioni

ELEMENTO TRAVE/2

3D

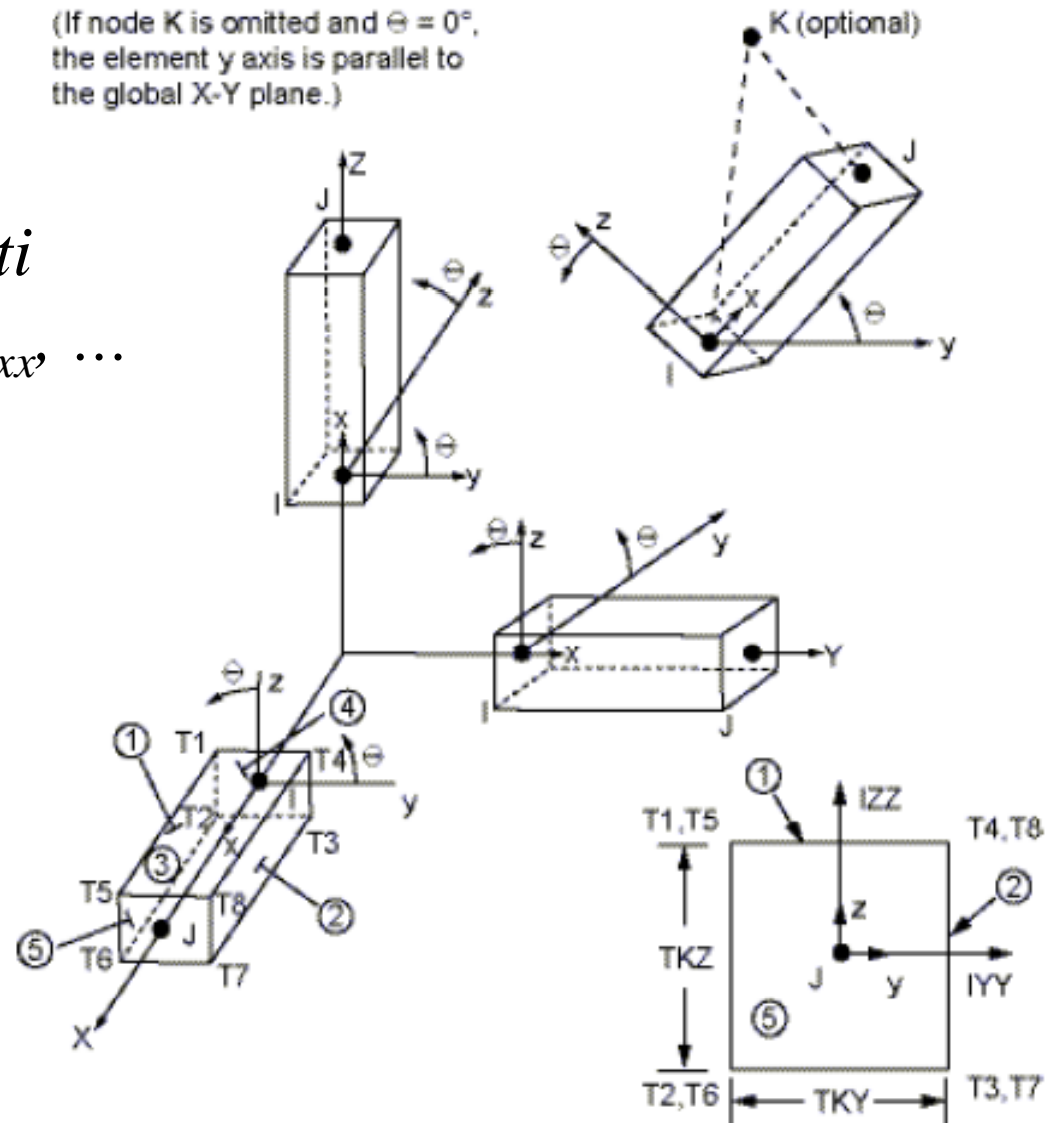
Telai spaziali

- 2 (3) nodi
- 6 g.d.l /nodo
- carichi concentrati e distribuiti
- Car. geometriche: A , J_{zz} , J_{yy} , J_{xx} ...

- Il SR di elemento è definito per convenzione o con il 3° nodo
- Gli assi “y” e “z” locali **devono** coincidere con gli assi principali di inerzia della sezione

Figure 4.1 BEAM4 Geometry

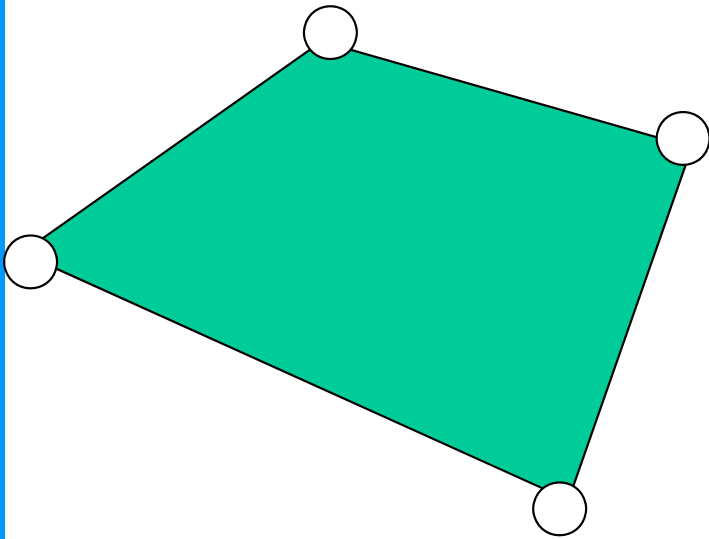
(If node K is omitted and $\Theta = 0^\circ$, the element y axis is parallel to the global X-Y plane.)





ELEMENTO TRAVE/3

Elementi piani: ogni nodo rappresenta un punto del continuo, tramite due g.d.l.

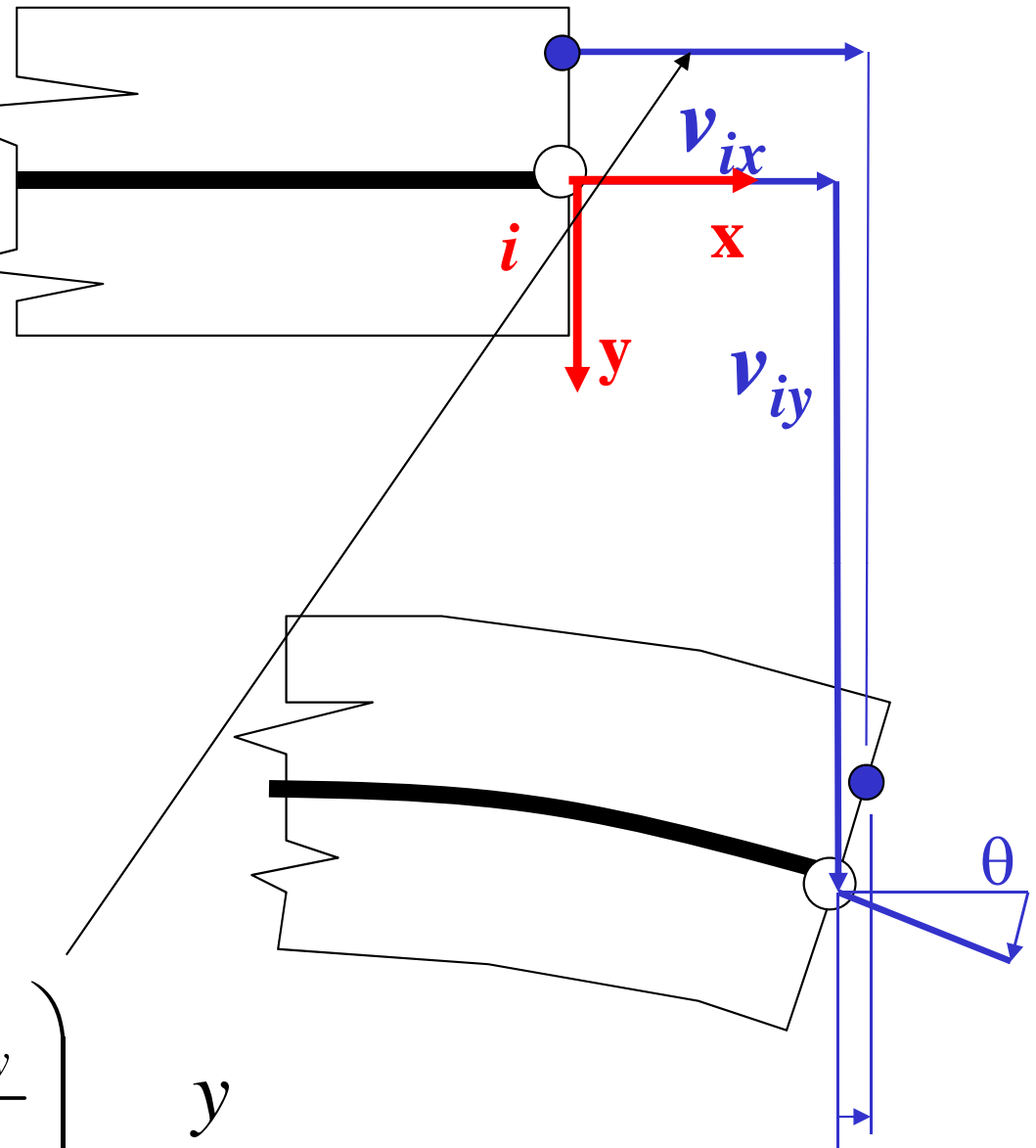


ELEMENTO TRAVE/3

Trave: con il nodo si vuole rappresentare lo stato di spostamento dell'intera sezione

Ipotesi sezioni piane

3 g.d.l. per nodo



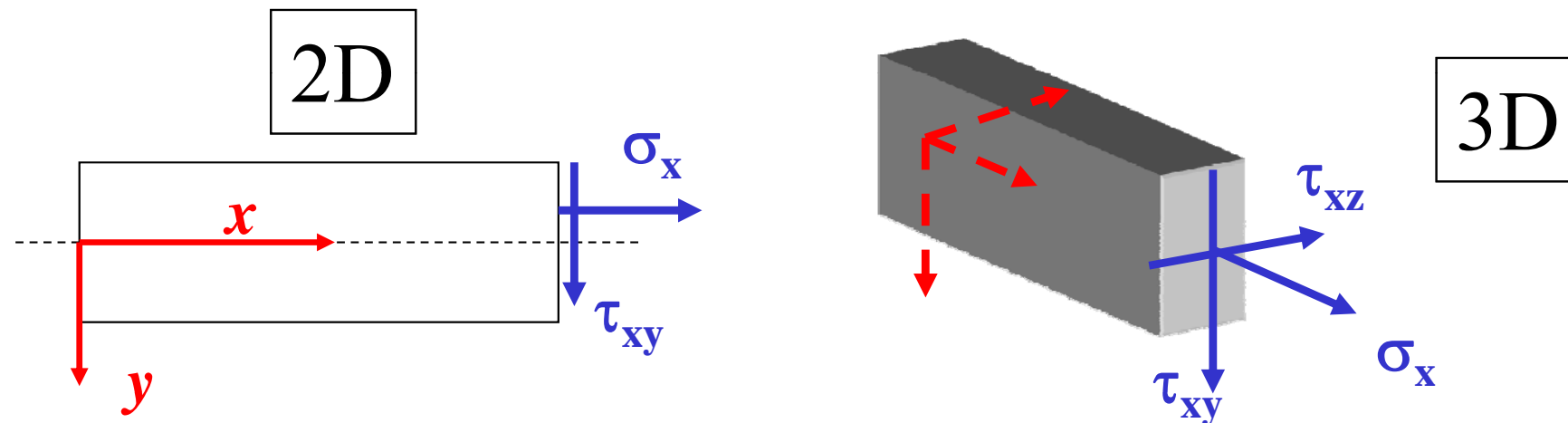
$$v_x(y) = v_{ix} + \theta y = v_{ix} - \left(\frac{\partial v_y}{\partial x} \right)_{x=x_i} y$$

$$\theta y = - \left(\frac{\partial v_y}{\partial x} \right)_{x=x_i} y$$

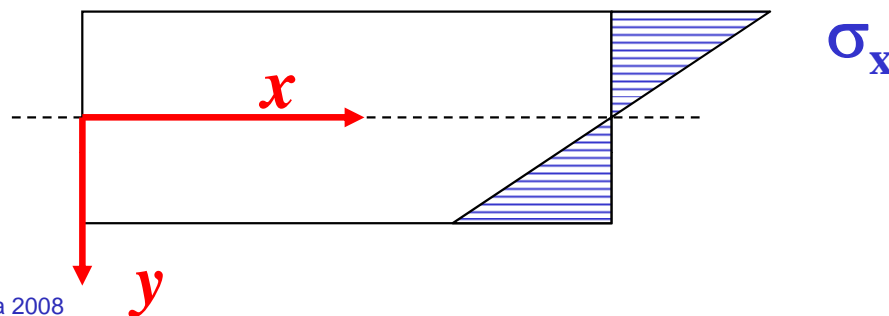
ELEMENTO TRAVE/4

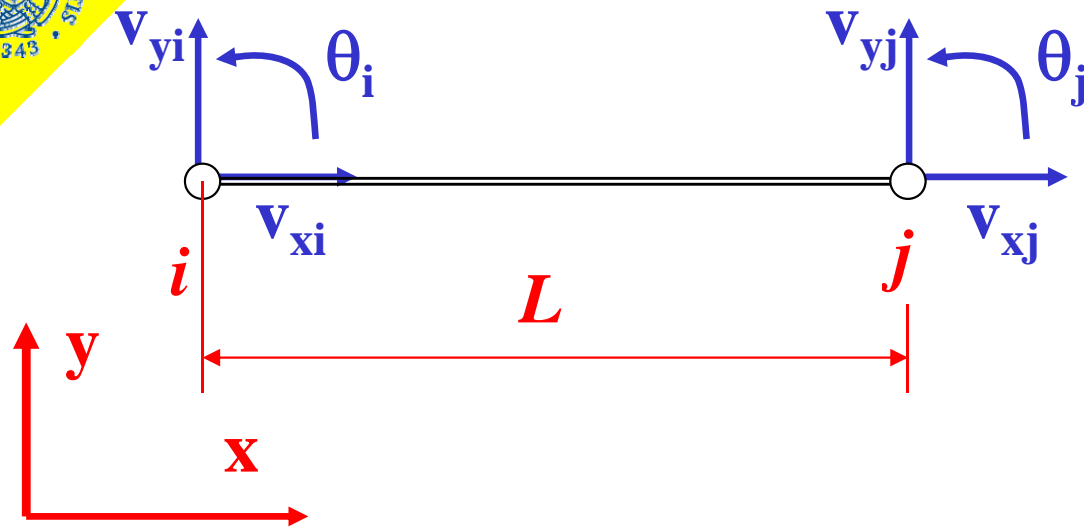
Stato di tensione/deformazione implicitamente conseguente alla scelta di elementi trave:

- le deformazioni dovute al taglio sono trascurate
- le uniche componenti di tensione non nulle sono:



- le σ_x hanno un andamento lineare nella sezione (formula di Navier)



**ELEMENTO TRAVE/5**

$$\{v(x)\} = \begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} \quad \{U^e\} = \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

Piccoli spostamenti/deformazioni

$$v_x(x) = f(v_{ix}, v_{jx})$$

2 condizioni per $v_x(x)$

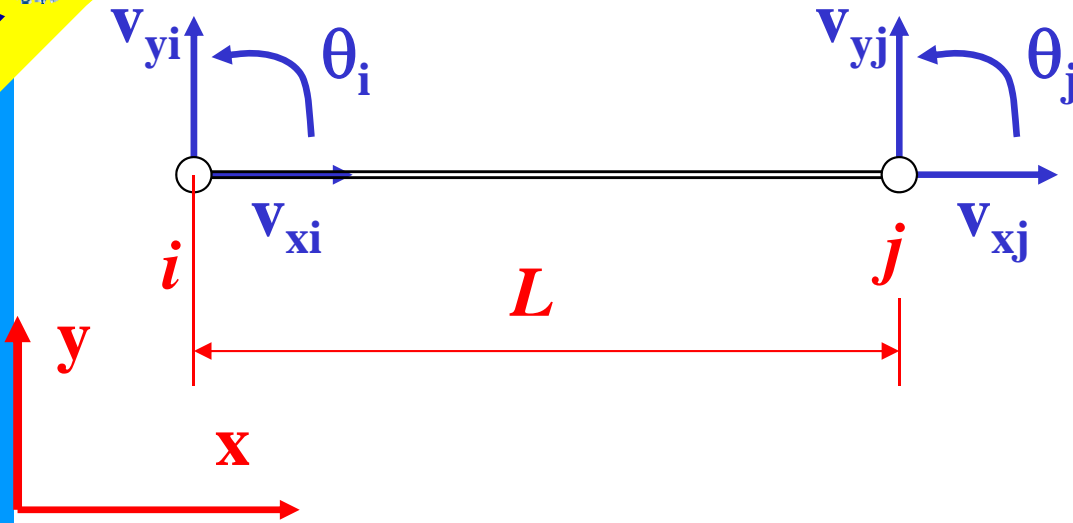
F.ni di forma lineari in "x"

$$v_x(x) = \left(1 - \frac{x}{L}\right) v_{ix} + \frac{x}{L} v_{jx} = N_{11} v_{ix} + N_{14} v_{jx}$$

$$\{v(x)\} = [N(x)] \{U^e\}$$

$3 \times 1 \quad 3 \times 6 \quad 6 \times 1$

$$N_{12} = N_{13} = N_{15} = N_{16} = 0$$



4 condizioni per $v_y(x)$

$v_y(x)$ di 3° grado in "x"

$$v_y(x) = A + Bx + Cx^2 + Dx^3$$

$$\theta = B + 2Cx + 3Dx^2$$

ELEMENTO TRAVE/6

$$\{v(x)\} = \begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} \quad \{U^e\} = \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

$$\theta = \frac{d v_y}{dx}$$

$$\begin{cases} v_y(0) = v_{iy} & \theta(0) = \theta_i \\ v_y(L) = v_{jy} & \theta(L) = \theta_j \end{cases}$$

$$\begin{aligned}
 v_y &= v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) + \\
 &+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) \\
 \theta &= v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) + \\
 &+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)
 \end{aligned}$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$\begin{aligned}
 v_y &= v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) + \\
 &+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) \\
 \theta &= v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) + \\
 &+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)
 \end{aligned}$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$v_y = v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) +$$

$$+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right)$$

$$\theta = v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) +$$

$$+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} \mathbf{N}_{11} & \mathbf{0} & \mathbf{0} & \mathbf{N}_{14} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_{22} & \mathbf{N}_{23} & \mathbf{0} & \mathbf{N}_{25} & \mathbf{N}_{26} \\ \mathbf{0} & \mathbf{N}_{32} & \mathbf{N}_{33} & \mathbf{0} & \mathbf{N}_{35} & \mathbf{N}_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$v_y = v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) +$$

$$+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right)$$

$$\theta = v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) +$$

$$+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$v_y = v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) +$$

$$+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right)$$

$$\theta = v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) +$$

$$+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$v_y = v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) +$$

$$+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right)$$

$$\theta = v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) +$$

$$+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$v_y = v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) +$$

$$+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right)$$

$$\theta = v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) +$$

$$+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)$$

$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

$$v_y = v_{iy} \left(1 - 3 \left(\frac{x}{L} \right)^2 + 2 \left(\frac{x}{L} \right)^3 \right) + \theta_i \left(x - 2L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right) +$$

$$+ v_{jy} \left(3 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right)^3 \right) + \theta_j \left(-L \left(\frac{x}{L} \right)^2 + L \left(\frac{x}{L} \right)^3 \right)$$

$$\theta = v_{iy} \frac{1}{L} \left(-6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right) + \theta_i \left(1 - 4 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right) +$$

$$+ v_{jy} \frac{1}{L} \left(6 \left(\frac{x}{L} \right) - 6 \left(\frac{x}{L} \right)^2 \right) + \theta_j \left(-2 \left(\frac{x}{L} \right) + 3 \left(\frac{x}{L} \right)^2 \right)$$

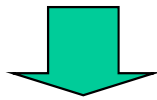
$$\begin{Bmatrix} v_x \\ v_y \\ \theta \end{Bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & N_{14} & 0 & 0 \\ 0 & N_{22} & N_{23} & 0 & N_{25} & N_{26} \\ 0 & N_{32} & N_{33} & 0 & N_{35} & N_{36} \end{bmatrix} \begin{Bmatrix} v_{ix} \\ v_{iy} \\ \theta_i \\ v_{jx} \\ v_{jy} \\ \theta_j \end{Bmatrix}$$

ELEM. TRAVE/7

ELEMENTO TRAVE/8

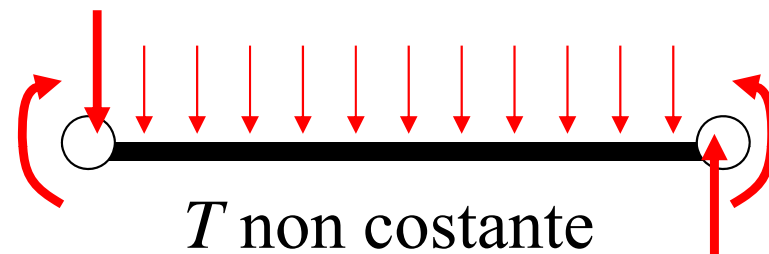
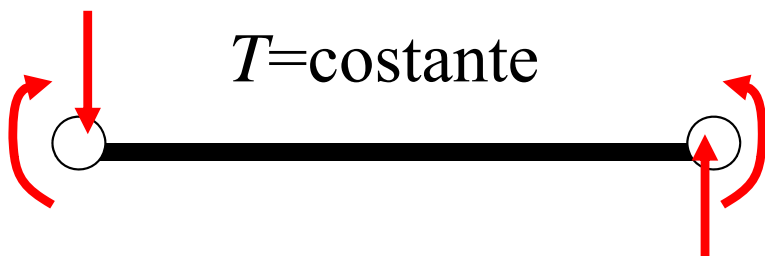
Oss.ne: la f.ne utilizzata per rappresentare la deformata della trave è una cubica.

$$v_y(x) = A + Bx + Cx^2 + Dx^3$$



$$T_y = \frac{d^3 v_y(x)}{dx^3} = \text{costante}$$

Le f.ni di forma rappresentano correttamente punto per punto la deformata del tratto di trave solo nel caso di taglio costante. Negli altri casi la rappresentazione di spostamenti, deformazioni e tensioni nei punti interni è approssimata, con errore che decresce al diminuire delle dimensioni dell'elemento





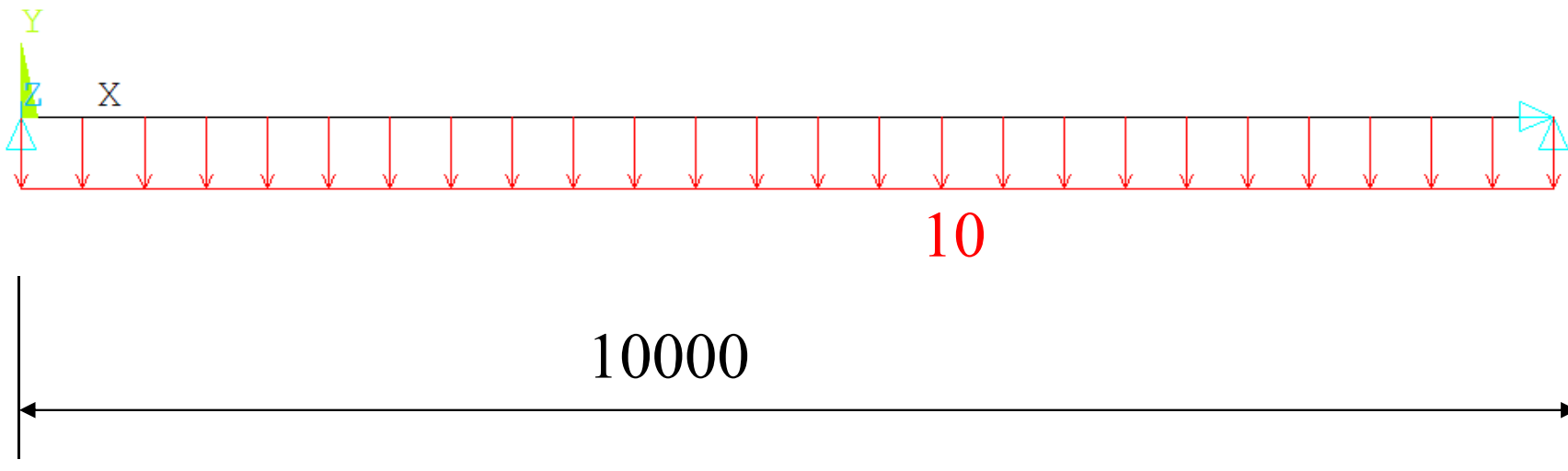
ELEMENTO TRAVE/9

Esempio: trave appoggiata con carico uniformemente distribuito

$$A=10^4$$

$$J=10^8$$

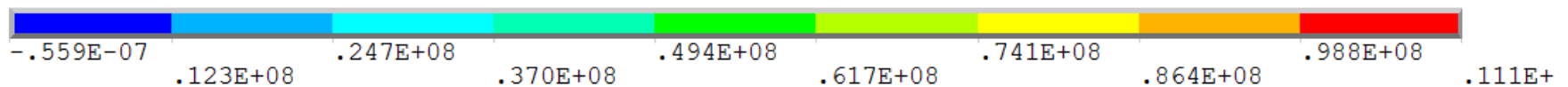
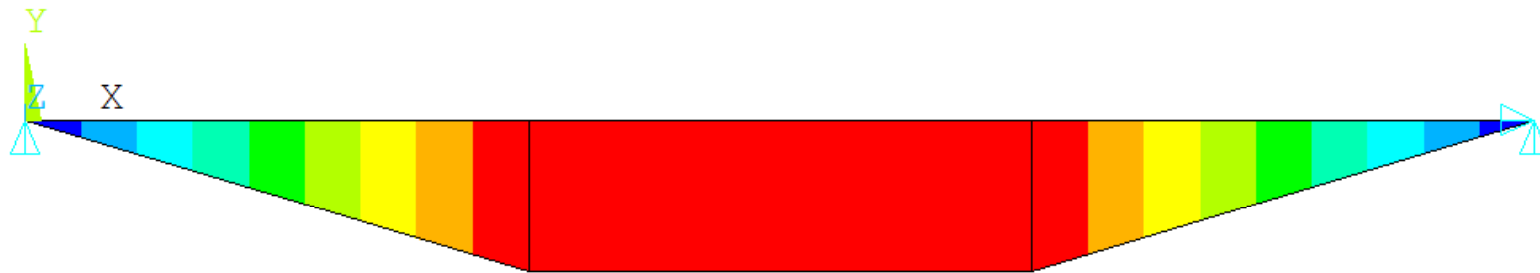
$$M_{\max} = 1.25 \cdot 10^8$$



ELEMENTO TRAVE/9

Esempio: trave appoggiata con carico uniformemente distribuito

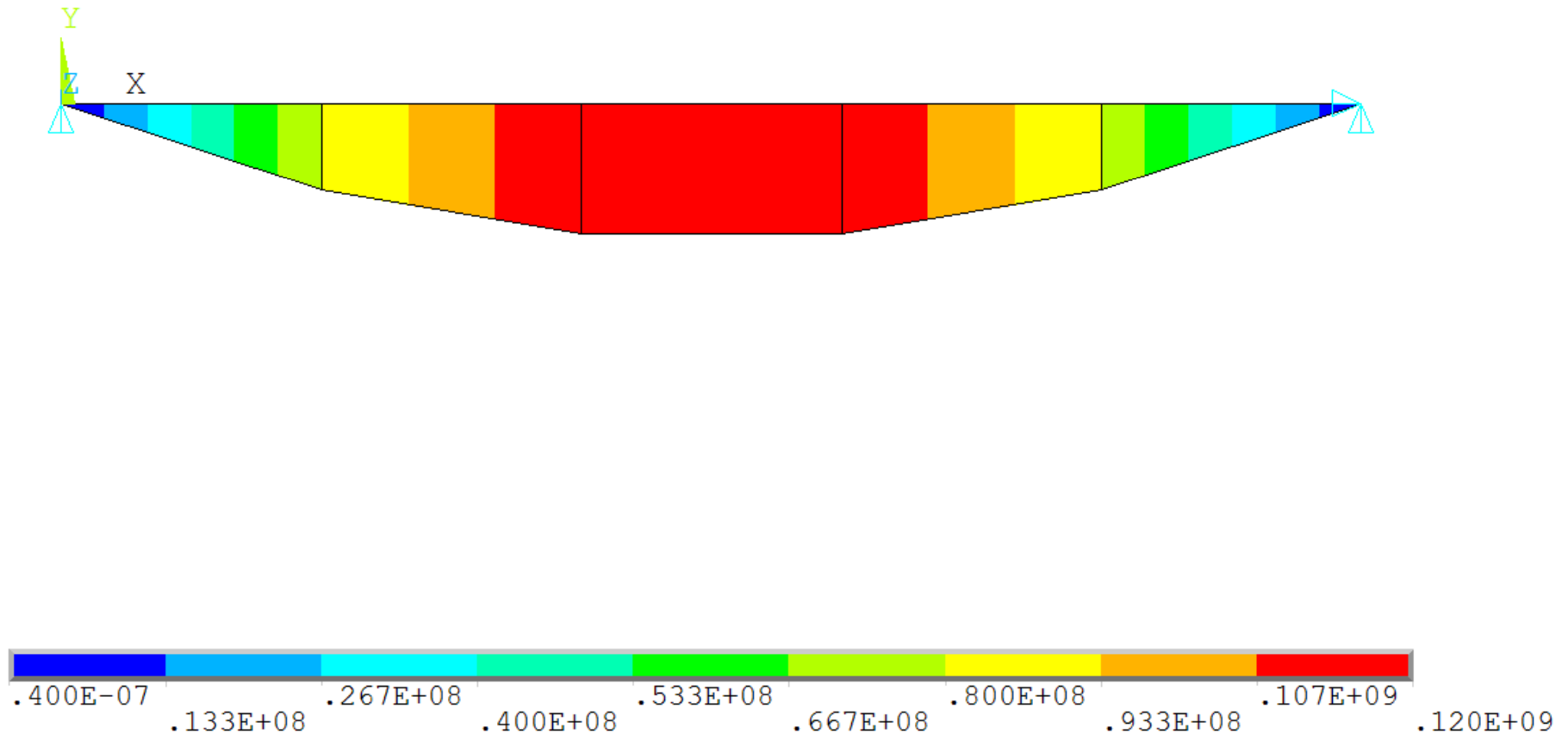
3 ELEMENTI – ERRORE = 11%



ELEMENTO TRAVE/9

Esempio: trave appoggiata con carico uniformemente distribuito

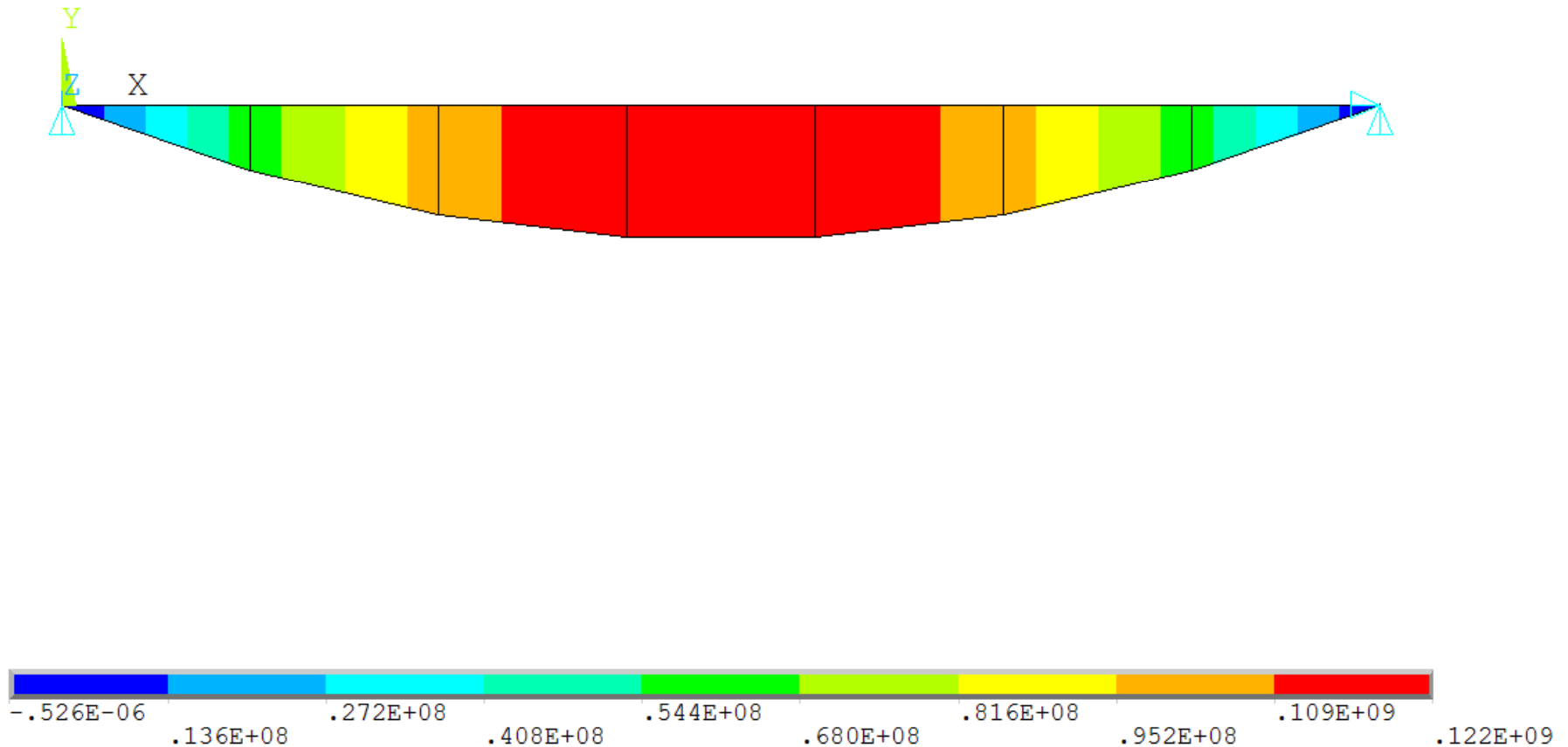
5 ELEMENTI – ERRORE = 4%



ELEMENTO TRAVE/9

Esempio: trave appoggiata con carico uniformemente distribuito

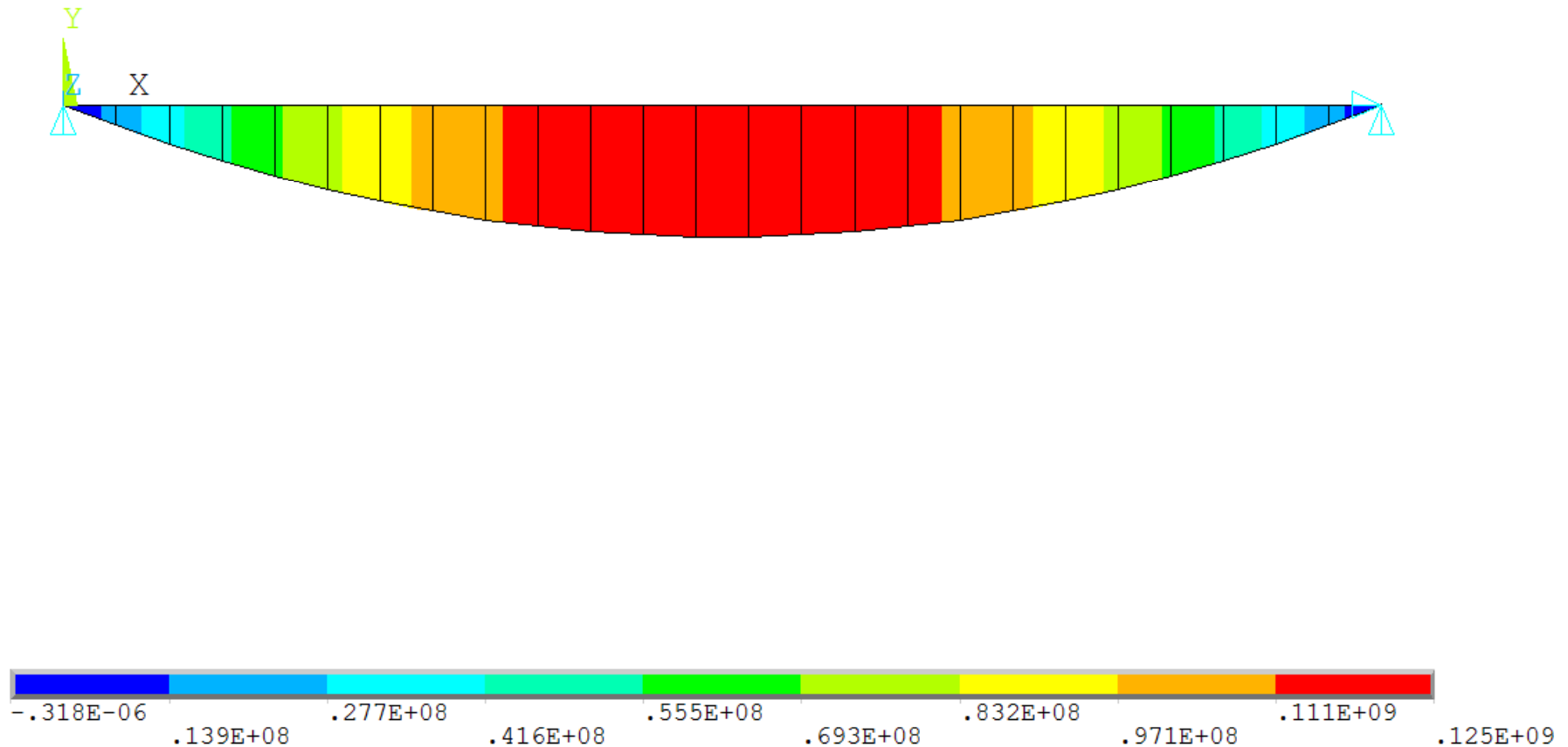
7 ELEMENTI – ERRORE = 2.4%



ELEMENTO TRAVE/9

Esempio: trave appoggiata con carico uniformemente distribuito

25 ELEMENTI – ERRORE $\approx 0\%$



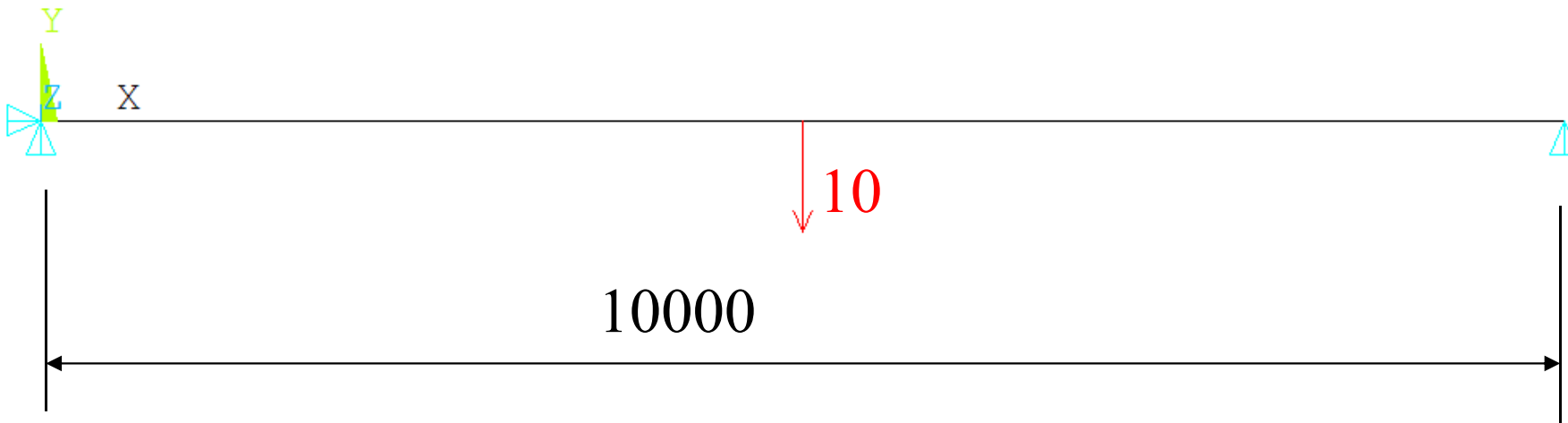
ELEMENTO TRAVE/10

Esempio: trave appoggiata con carico concentrato

$$A=10^4$$

$$J=10^8$$

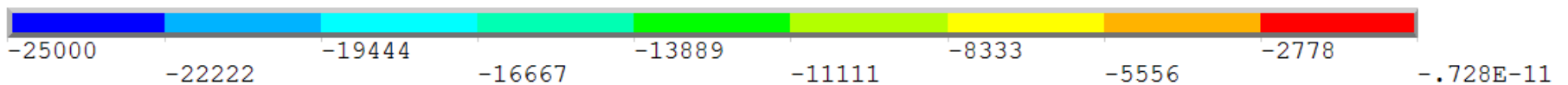
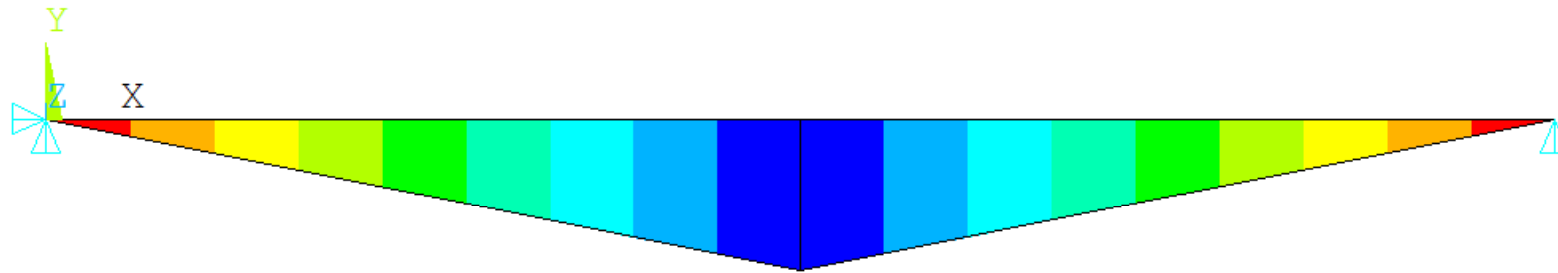
$$M_{\max} = 2.5 \cdot 10^4$$



ELEMENTO TRAVE/10

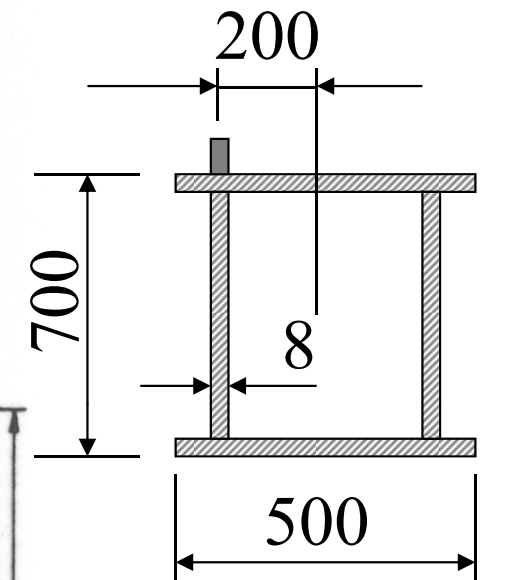
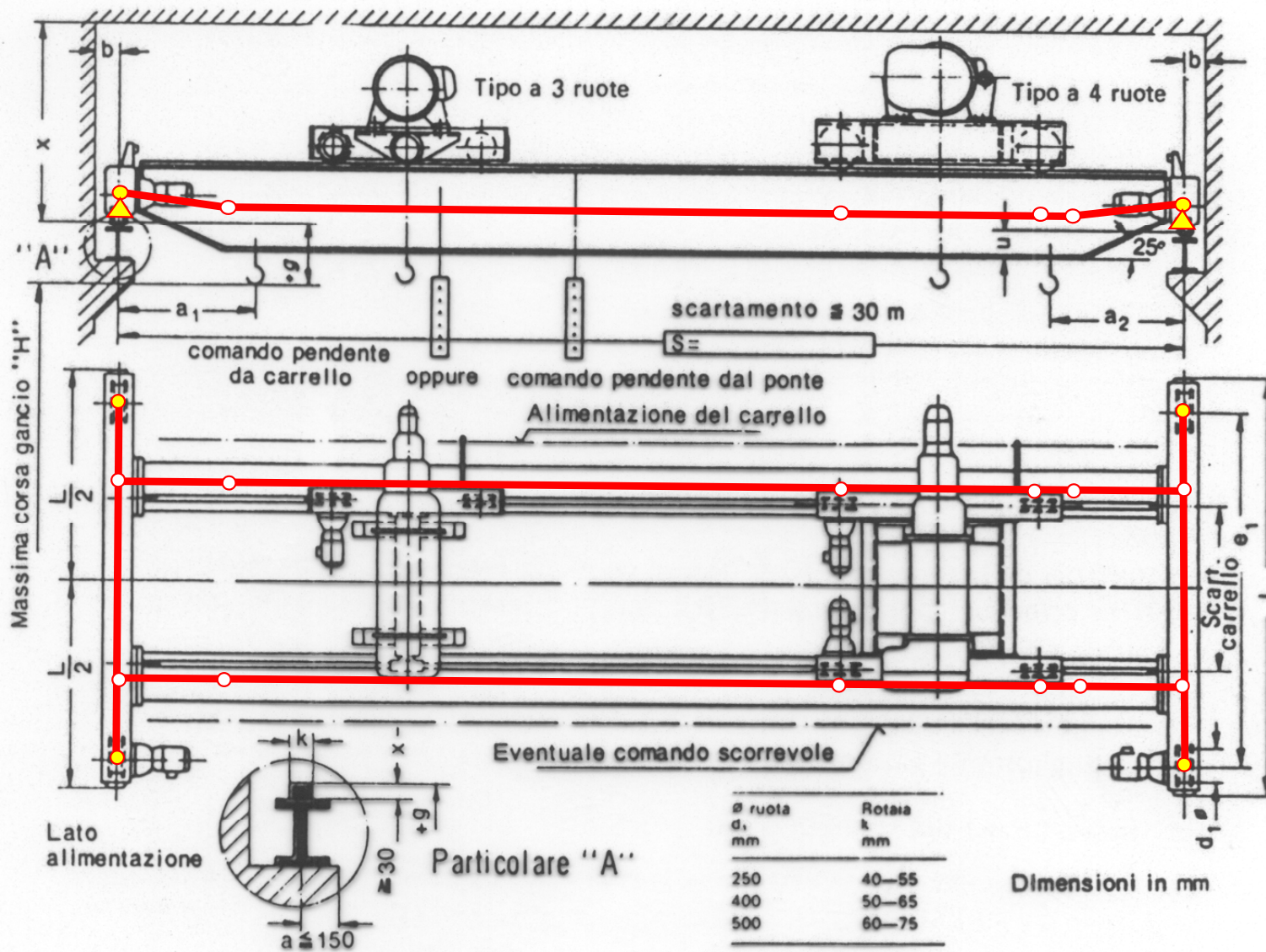
Esempio: trave appoggiata con carico concentrato

2 ELEMENTI – ERRORE = 0 %

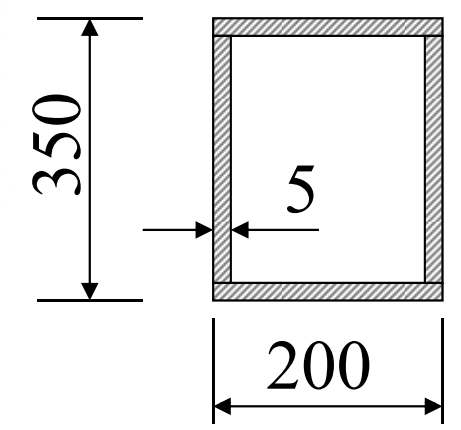


ELEMENTO TRAVE/11 – GRU A PONTE

3D



Trave principale



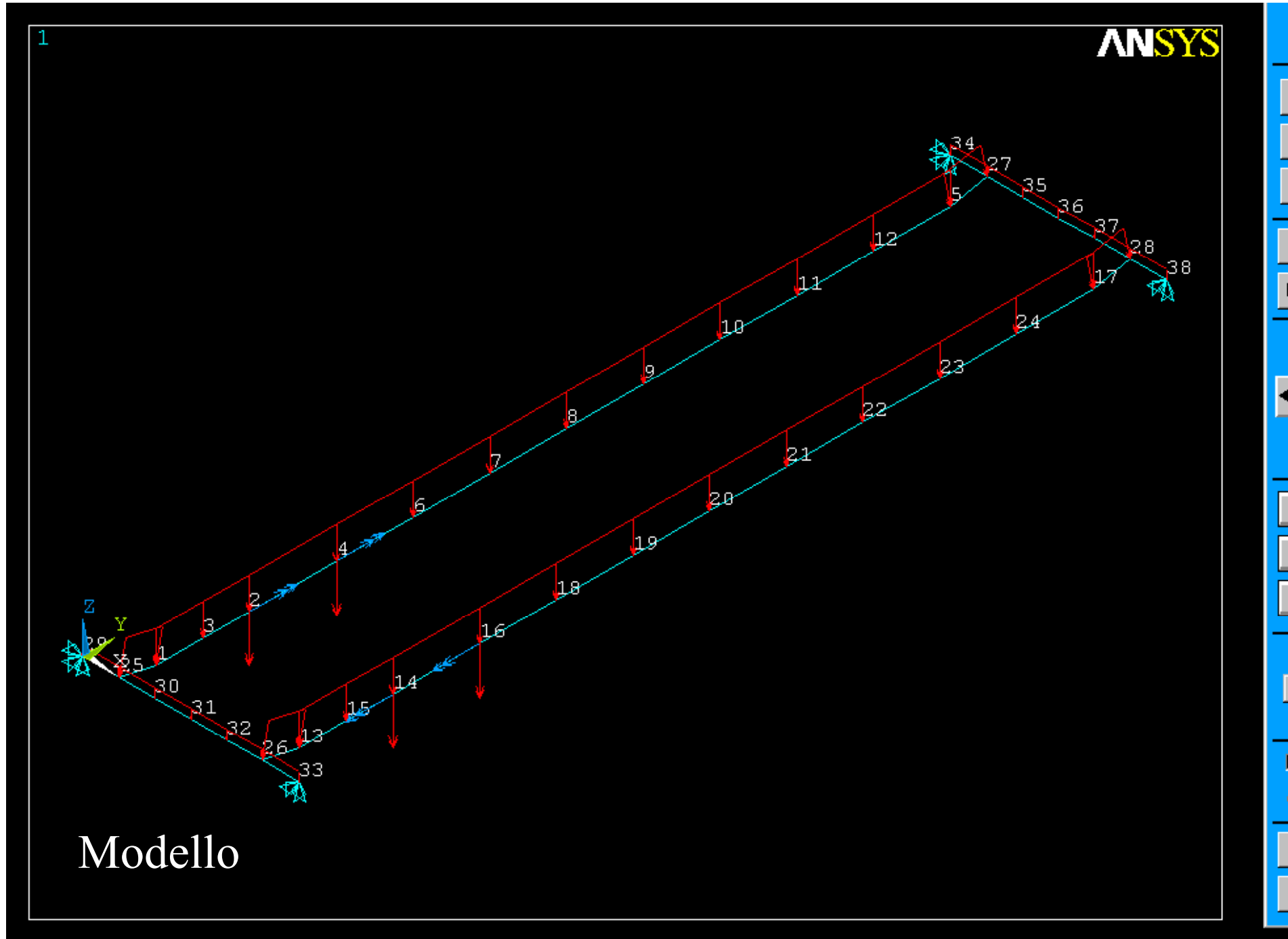
Testata

Interasse ruote testata (e_1) = 5 m
 Scartamento (S) = 20 m
 Scartamento carrello = 2.5 m

ELEMENTO TRAVE/12 – GRU A PONTE

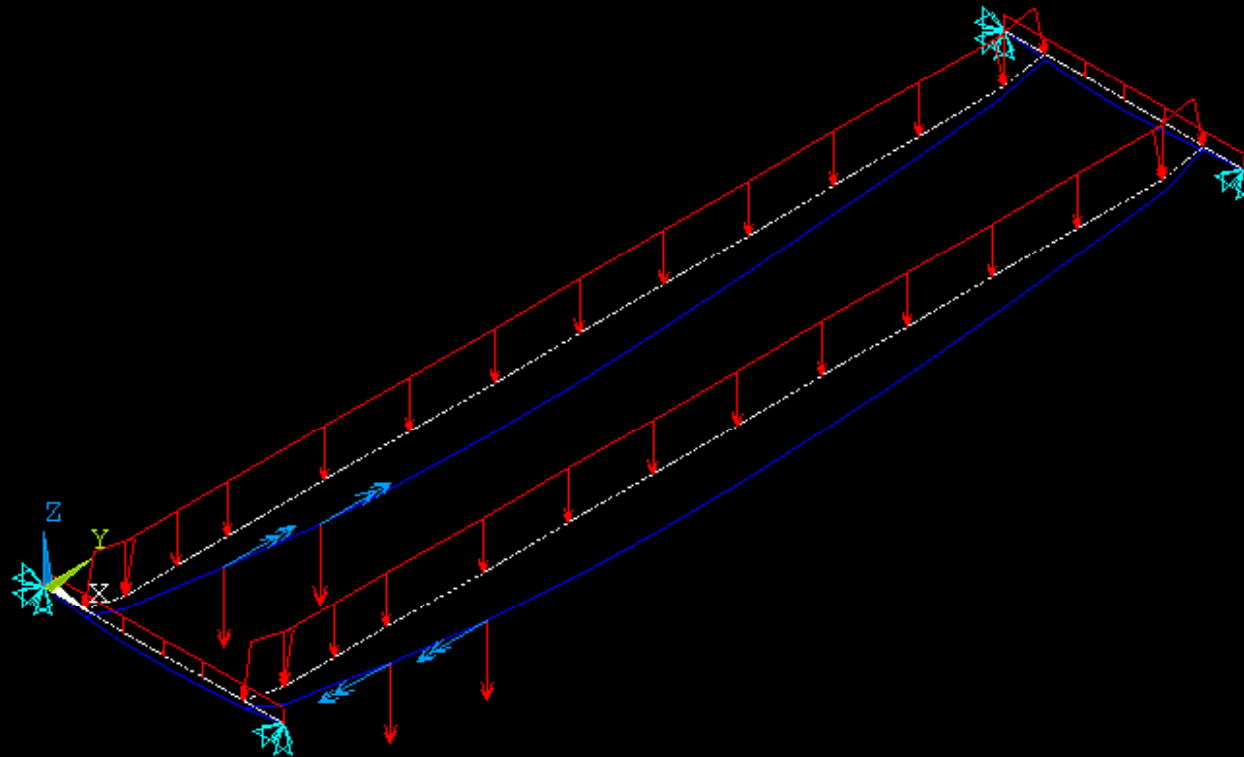
Scuola di Dottorato in Ingegneria “Leonardo da Vinci”

File di comandi : GRU_A_PONTE.txt



ELEMENTO TRAVE/12 – GRU A PONTE

1



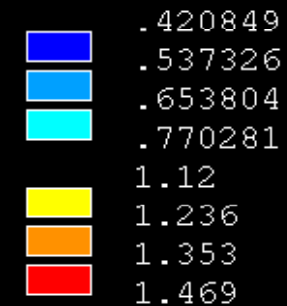
Deformata

```

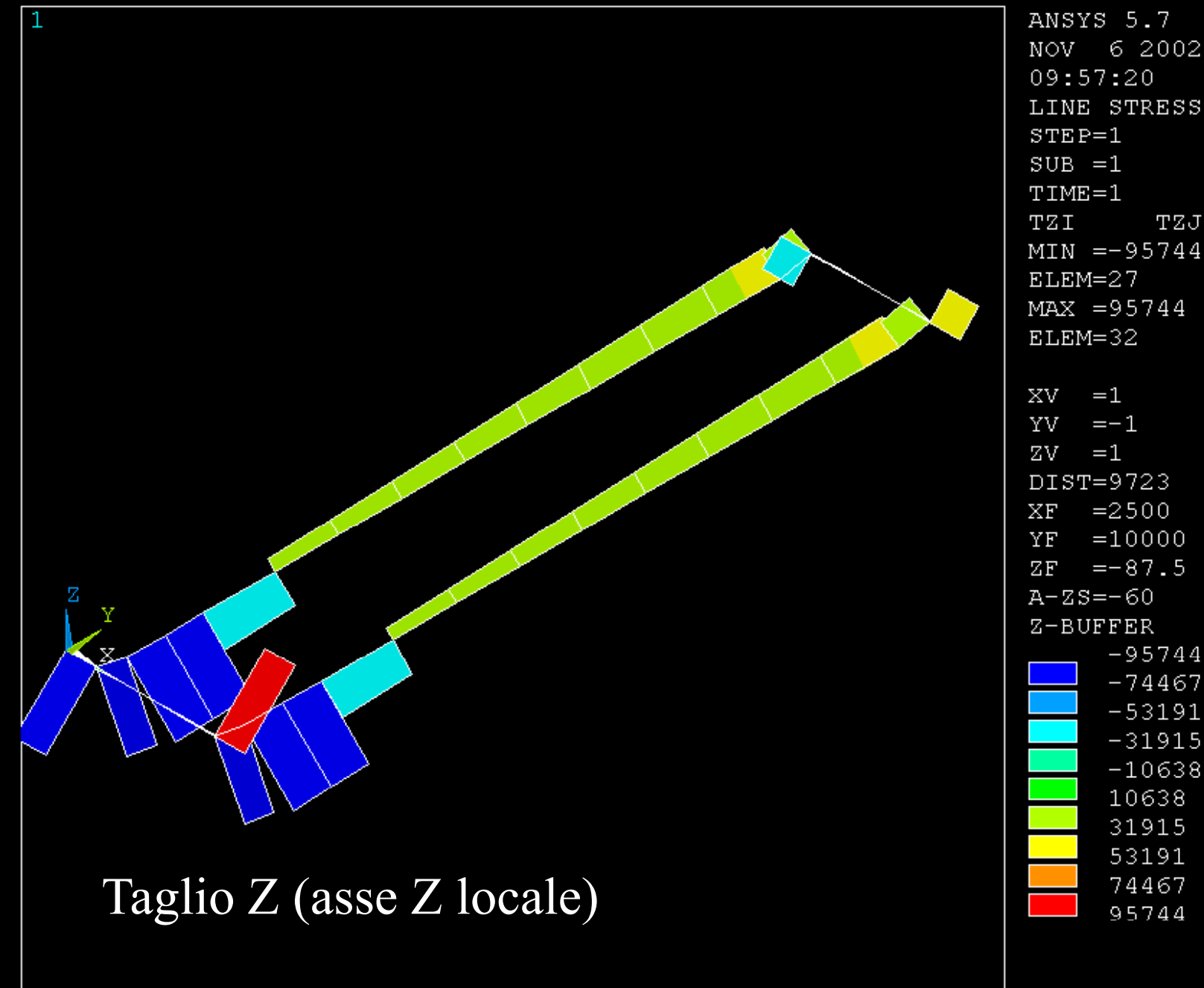
ANSYS 5.7
NOV  6 2002
09:56:33
DISPLACEMENT
STEP=1
SUB  =1
TIME=1
PowerGraphics
EFACET=1
AVRES=Mat
DMX  =48.486
U
F
M
    
```

```

DSCA=20.624
XV  =1
YV  =-1
ZV  =1
DIST=9723
XF  =2500
YF  =10000
ZF  =-584.594
A-ZS=-60
Z-BUFFER
PRES-NORM
    
```

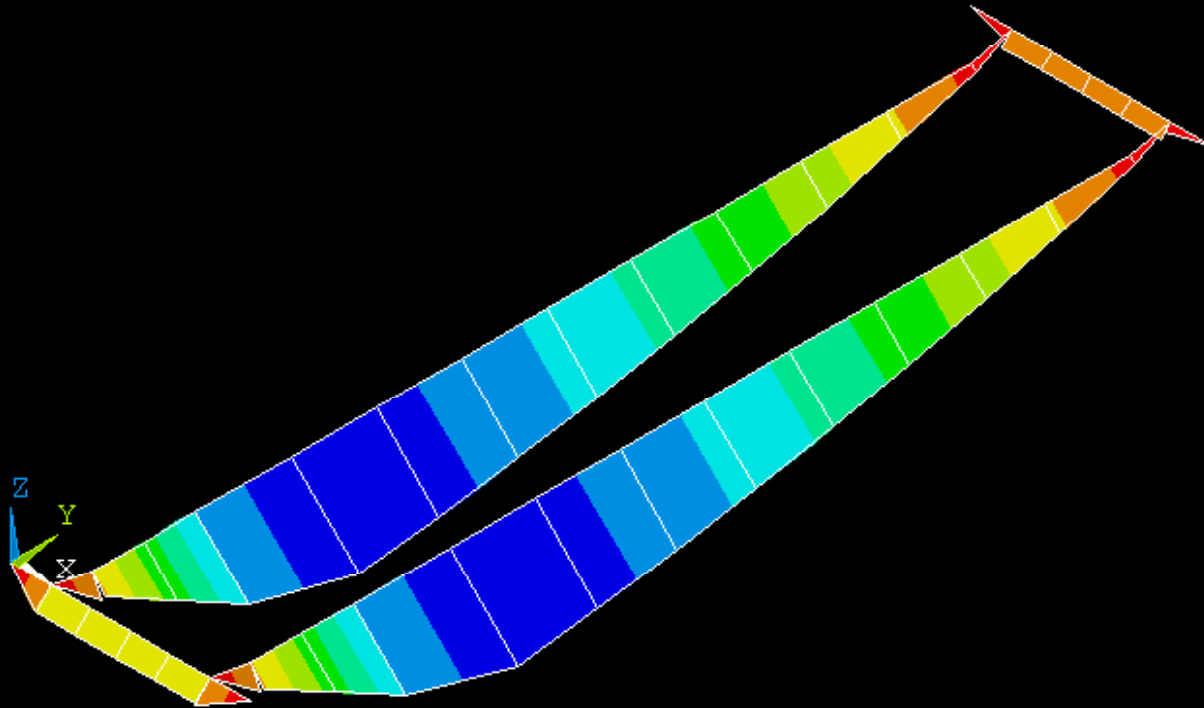


ELEMENTO TRAVE/12 – GRU A PONTE



ELEMENTO TRAVE/12 – GRU A PONTE

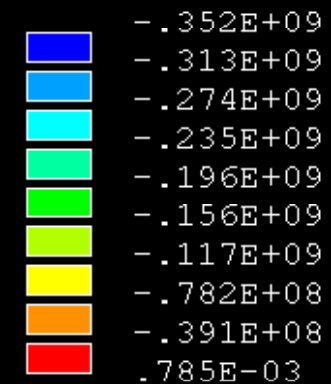
1



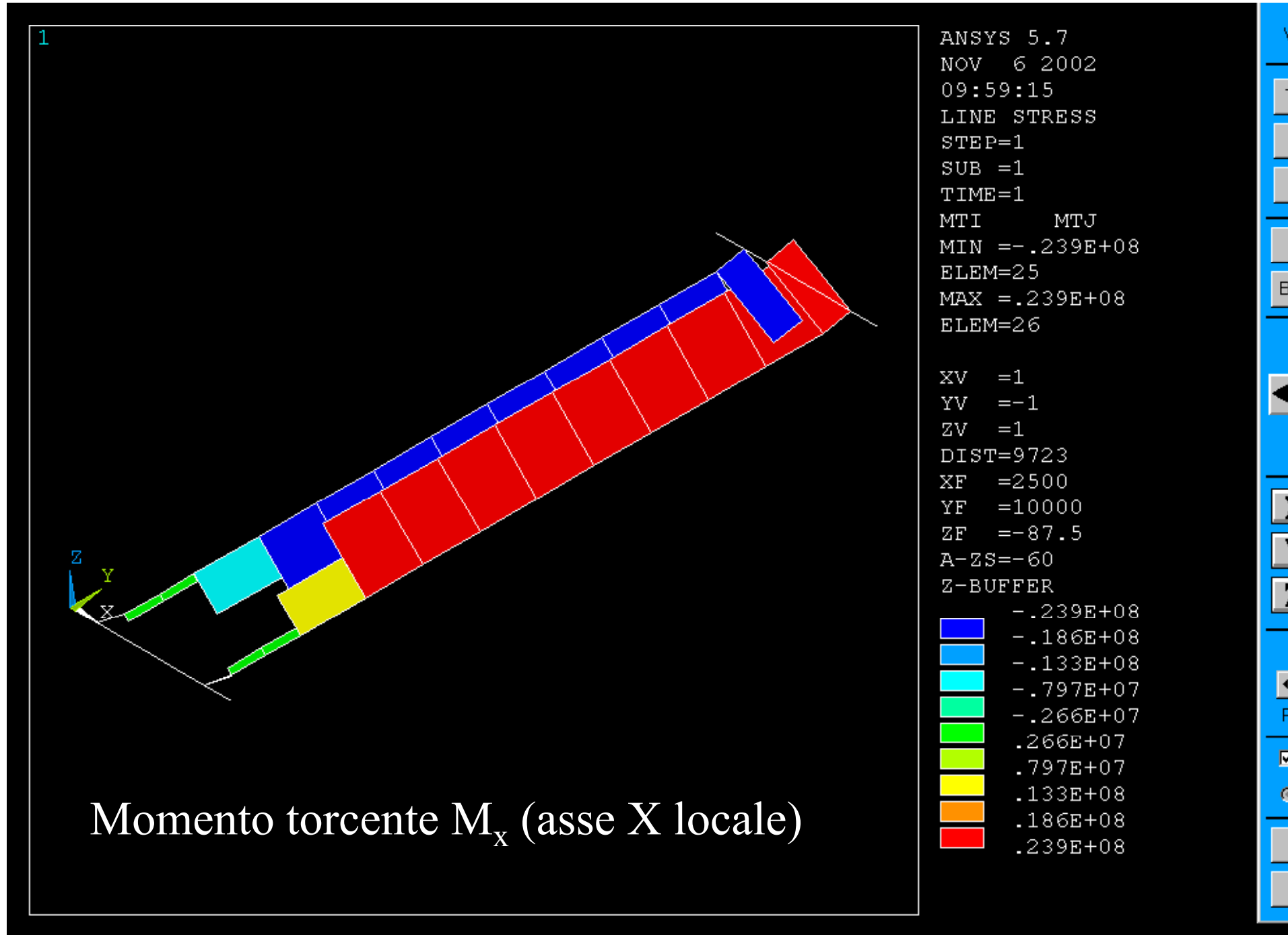
Momento flettente M_y (asse Y locale)

```
ANSYS 5.7
NOV  6 2002
09:58:51
LINE STRESS
STEP=1
SUB  =1
TIME=1
MYI      MYJ
MIN  =-.352E+09
ELEM=14
MAX  =.785E-03
ELEM=26
```

```
XV  =1
YV  =-1
ZV  =1
DIST=9723
XF  =2500
YF  =10000
ZF  =-87.5
A-ZS=-60
Z-BUFFER
```



ELEMENTO TRAVE/12 – GRU A PONTE





ELEMENTO TRAVE/13

Dati di input per l'elemento trave 2D (BEAM3) di ANSYS

Nodes

I, J

Degrees of Freedom

UX, UY, ROTZ

Real Constants

AREA - Cross-sectional area

IZZ - Area moment of inertia

HEIGHT - Total beam height

SHEARZ - Shear deflection constant

ISTRN - Initial strain

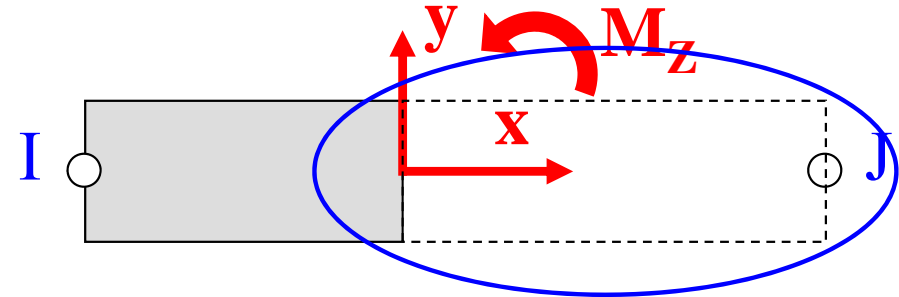
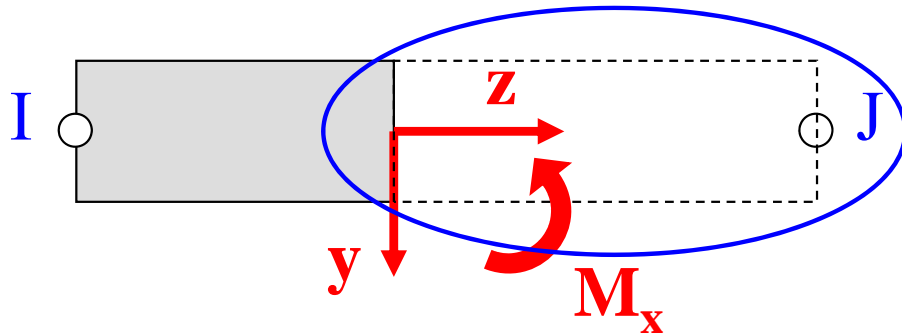
ADDMAS - Added mass per unit length

ELEMENTO TRAVE/14

Caratteristiche di sollecitazione per l'elemento trave 2D di ANSYS

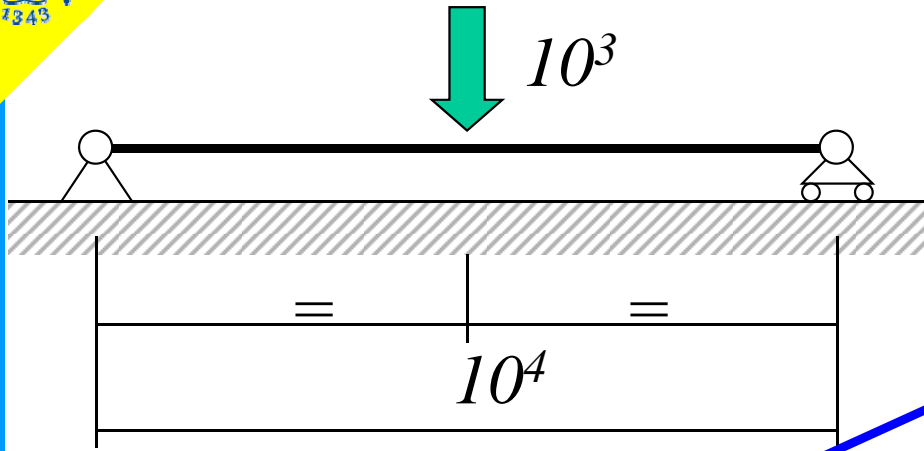
SR tradizionale per le caratteristiche di sollecitazione

SR usato da ANSYS per le caratteristiche di sollecitazione (Il SR è definito per ogni singolo elemento trave con senso di percorrenza dal nodo I al nodo J)

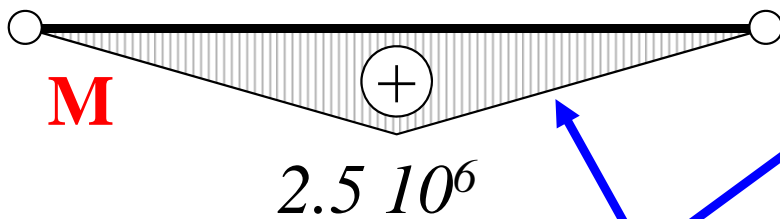
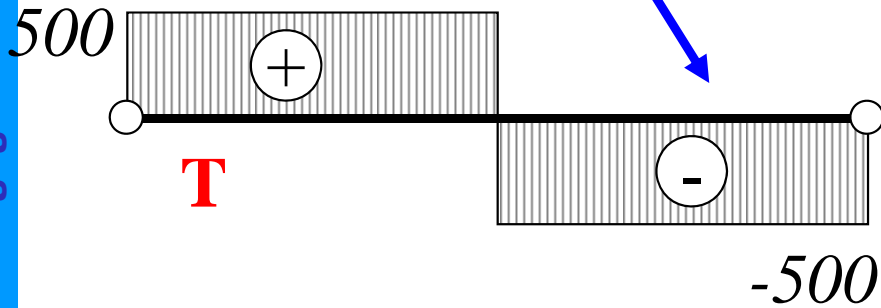


$$\begin{aligned} N &= \text{MFORX} \\ T_Y &= -\text{MFORY} \\ M_X &= \text{MMOMZ} \end{aligned}$$

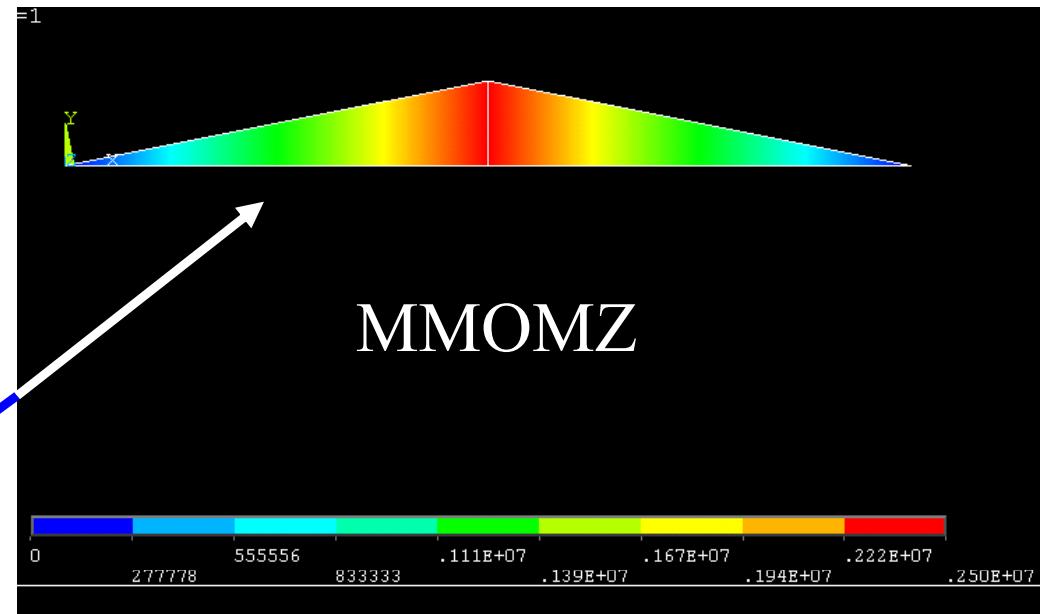
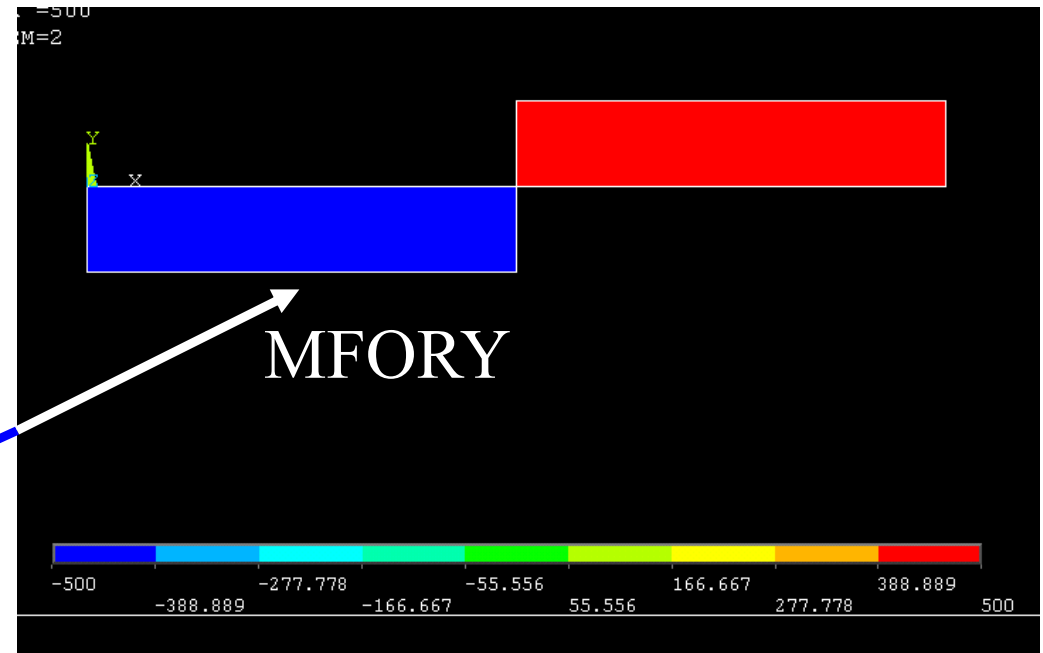
Risultanti di azioni agenti sul tratto di asta “a valle” della sezione



Segno del Taglio



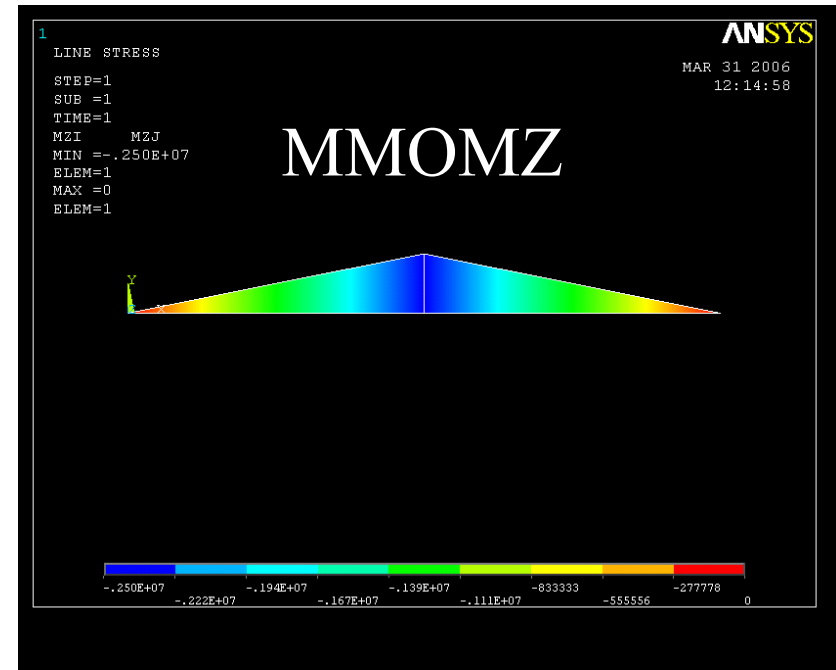
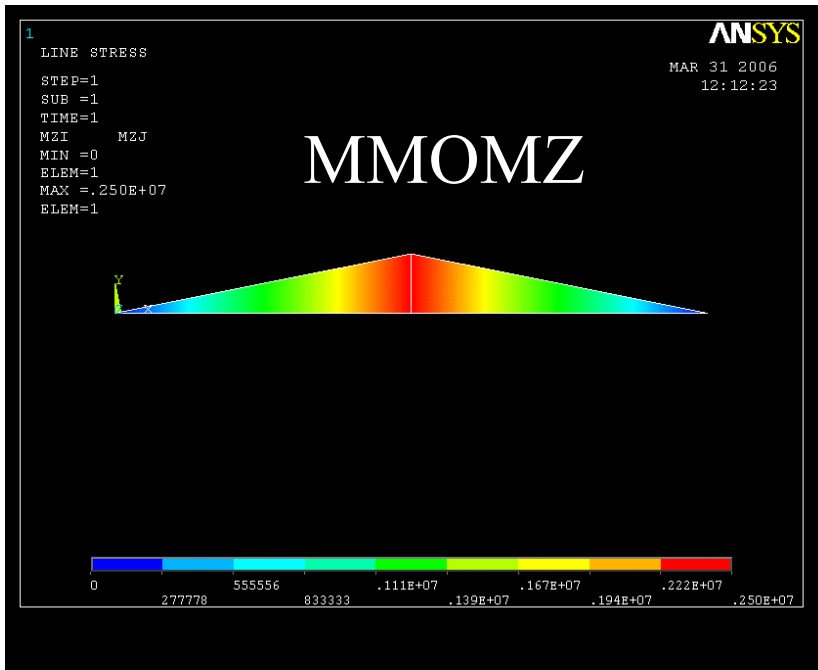
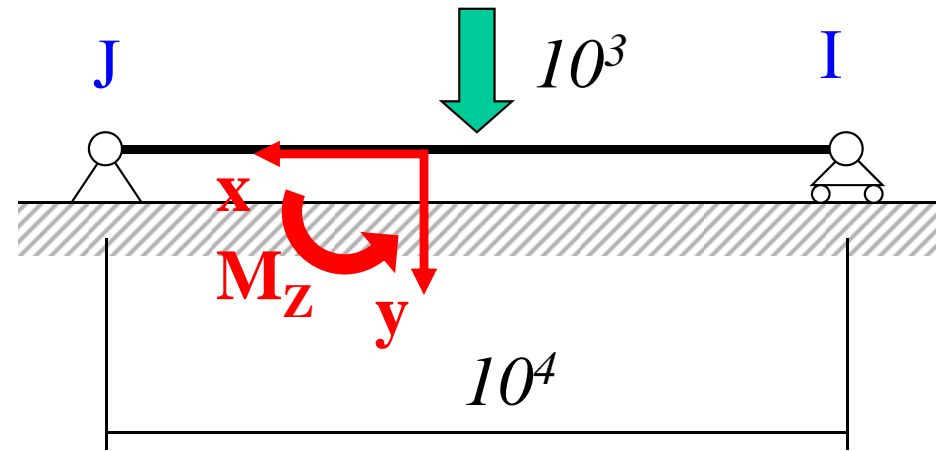
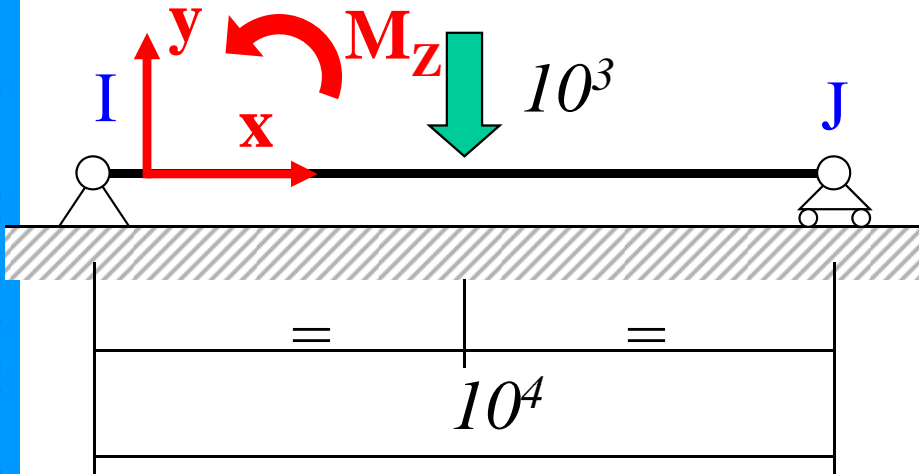
Posizione diagramma



ELEMENTO TRAVE/15

ELEMENTO TRAVE/16

Effetto del senso di percorrenza dell'elemento



ELEMENTO TRAVE/17

Dati di input per l'elemento trave 3D (BEAM4) di ANSYS

Nodes

I, J, K (K orientation node is optional)

Degrees of Freedom

UX, UY, UZ, ROTX, ROTY, ROTZ

Real Constants

AREA, IZZ, IYY, TKZ, TKY, THETA
ISTRN, IXX, SHEARZ, SHEARY, SPIN, ADDMAS

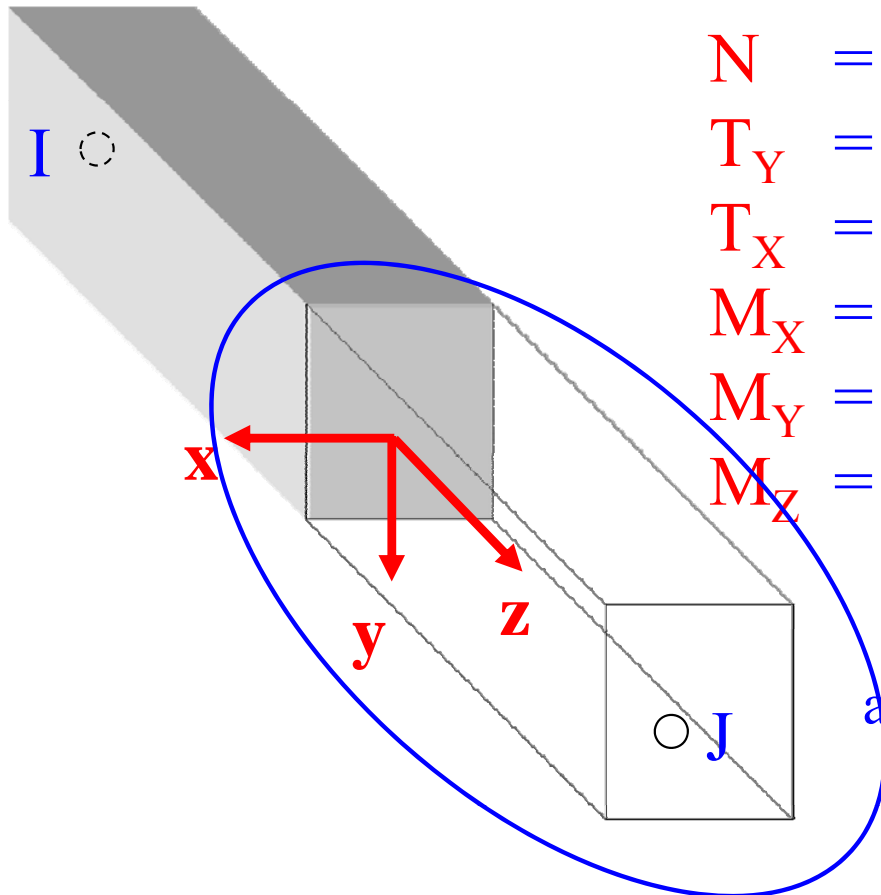
See [Table 4.1. "BEAM4 Real Constants"](#) for a description of the real constants.

Material Properties

EX, ALPX (or CTEX or THSX), DENS, GXY, DAMP

ELEMENTO TRAVE/18

SR tradizionale per le caratteristiche di sollecitazione



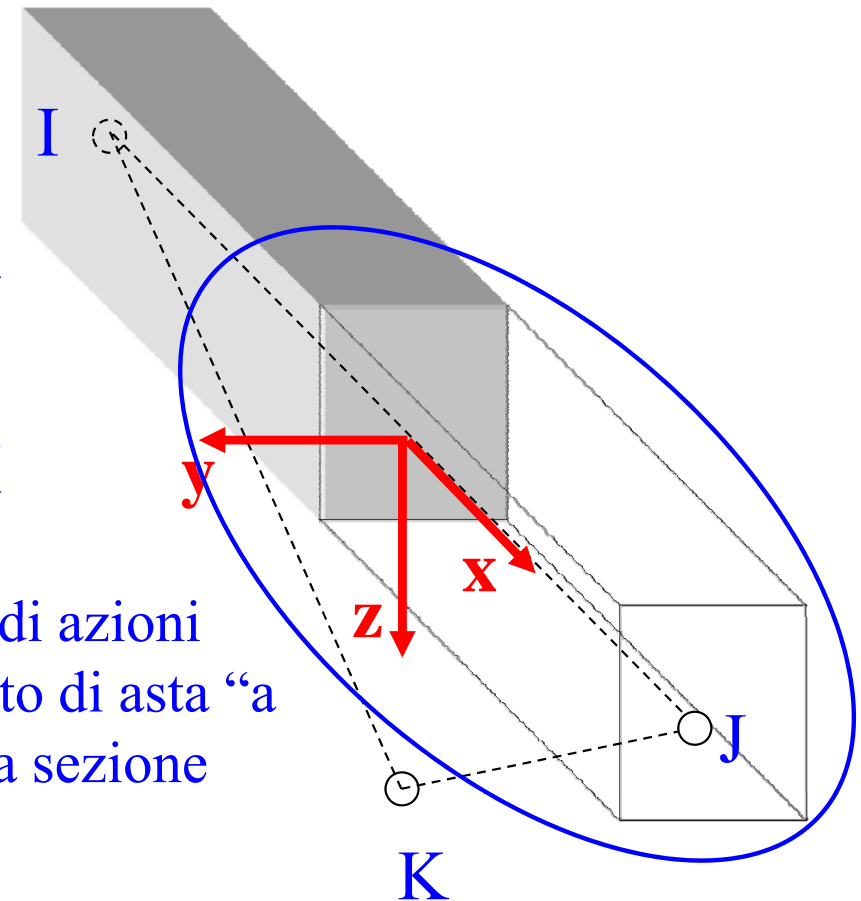
$$\begin{aligned}
 N &= \text{MFORX} \\
 T_Y &= \text{MFORZ} \\
 T_X &= \text{MFORY} \\
 M_X &= \text{MMOMY} \\
 M_Y &= \text{MMOMZ} \\
 M_Z &= \text{MMOMX}
 \end{aligned}$$

Risultanti di azioni agenti sul tratto di asta “a valle” della sezione

Car. sollecitazione trave 3D

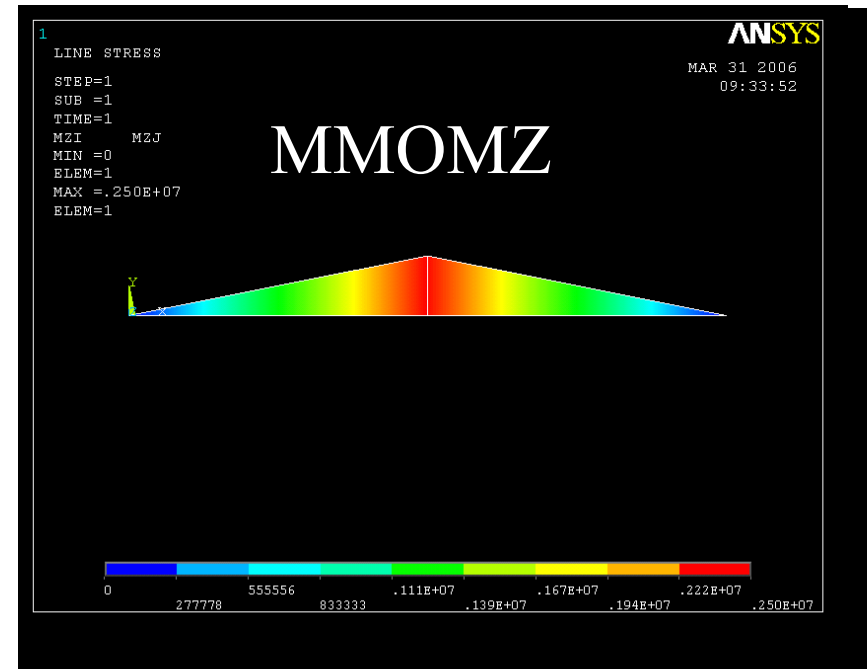
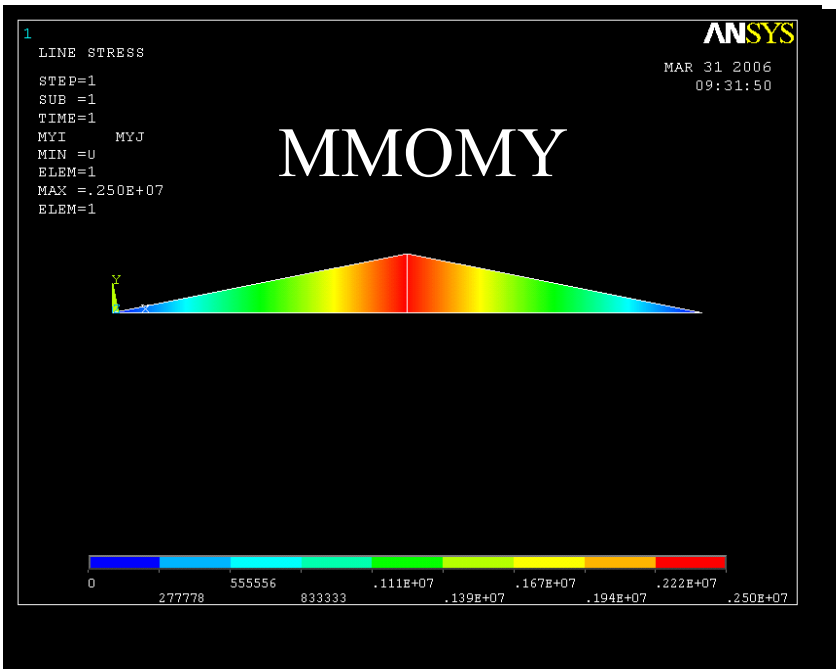
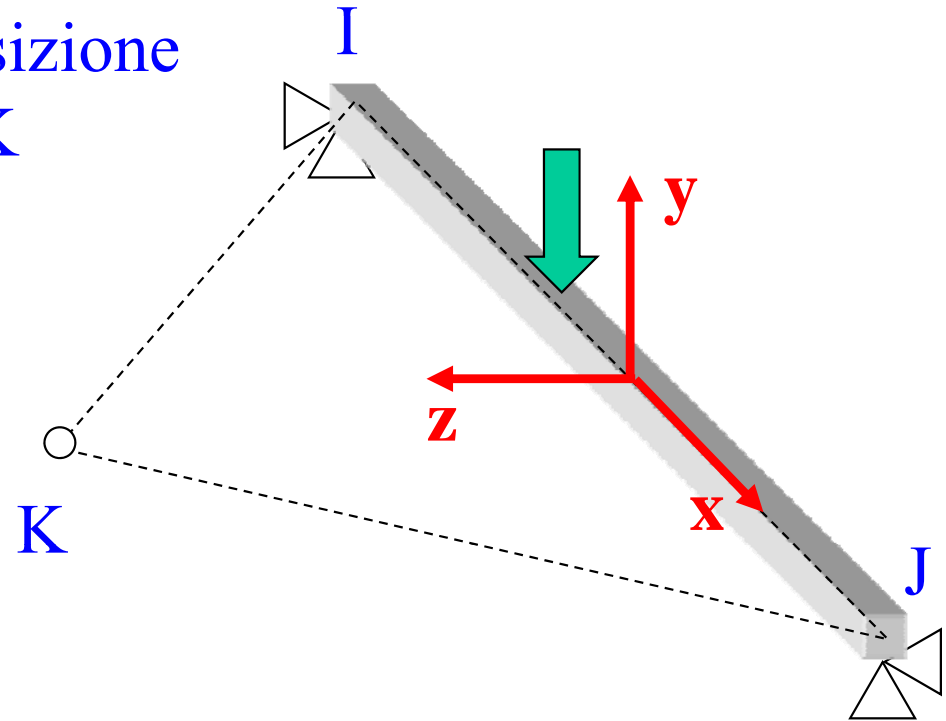
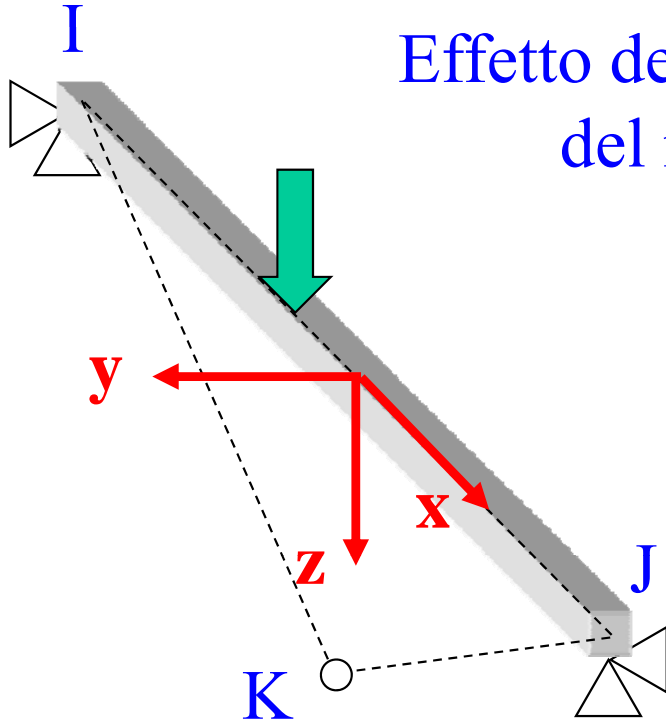
SR usato da ANSYS per le caratteristiche di sollecitazione

(Il SR è definito per ogni singolo elemento trave con senso di percorrenza dal nodo I al nodo J)



ELEMENTO TRAVE/19

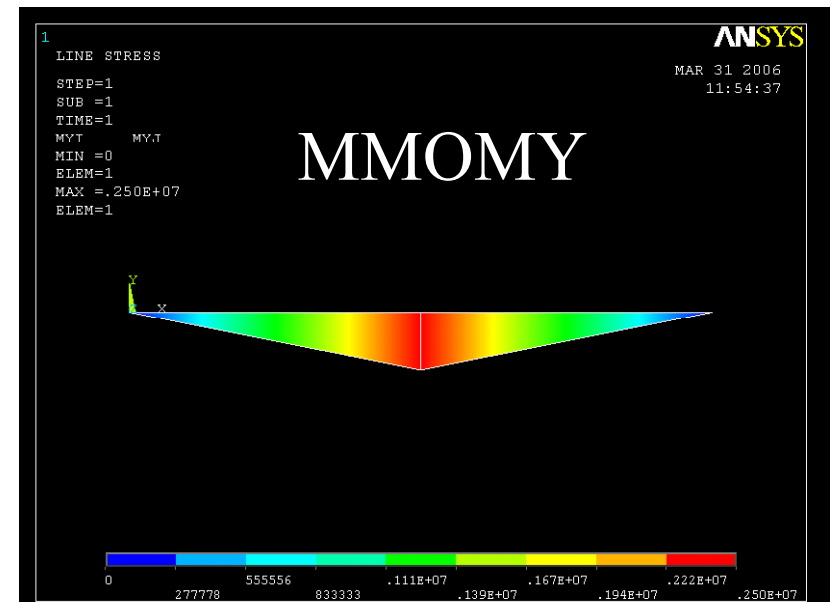
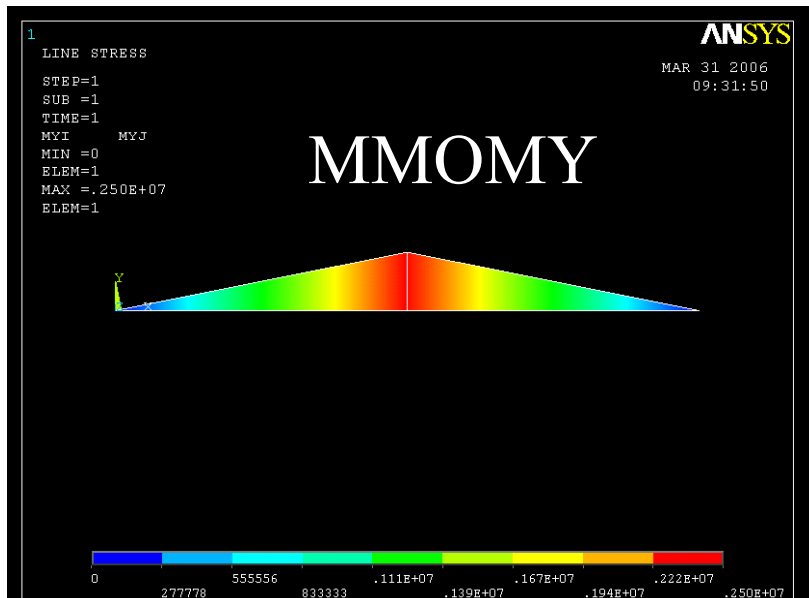
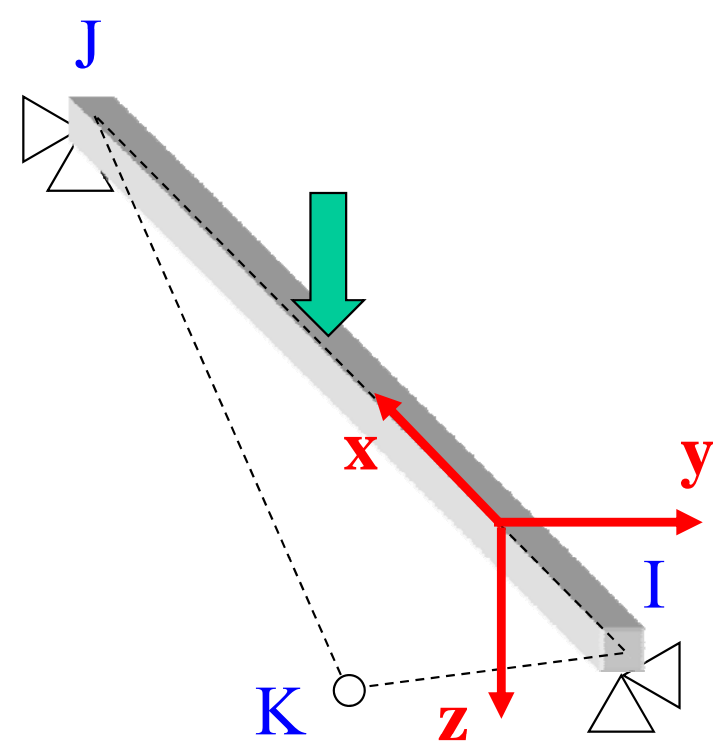
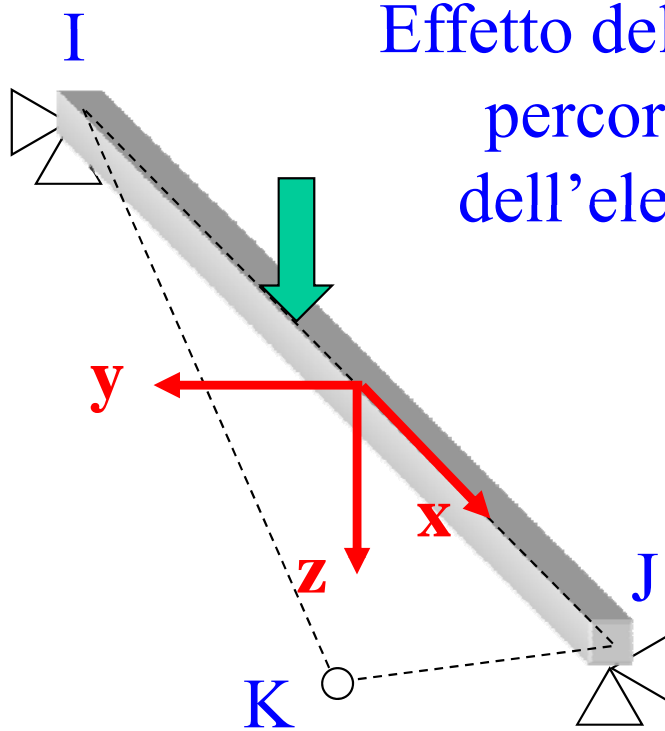
Effetto della posizione del nodo K





ELEMENTO TRAVE/20

Effetto del senso di percorrenza dell'elemento



ELEMENTO TRAVE/21

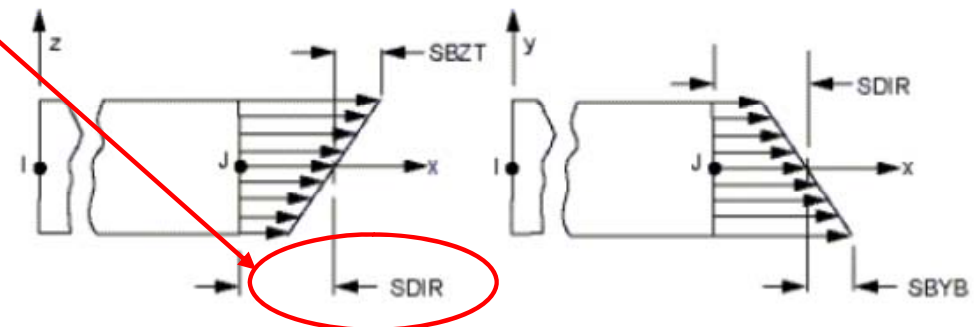
Accesso ai risultati per l'elemento trave 3D – Comando ETABLE

Table 4.3 BEAM4 Item and Sequence Numbers (KEYOPT(9) = 0)

Output Quantity Name	ETABLE and ESOL Command Input			
	Item	E	I	J
SDIR	LS	-	1	6
SBYT	LS	-	2	7
SBYB	LS	-	3	8
SBZT	LS	-	4	9
SBZB	LS	-	5	10
EDDEFIND	EDDEFI	-	1	6
EPFELBZB	LEPEL	-	5	10
SMAX	NMISC	-	1	3
SMIN	NMISC	-	2	4
EDDEFIND	EDDEFI	-	1	6

Tensione fibra baricentrica
 ETABLE,NI,LS,1 ! Nodo I
 ETABLE,NJ,LS,6 ! Nodo J

Figure 4.2 BEAM4 Stress Output

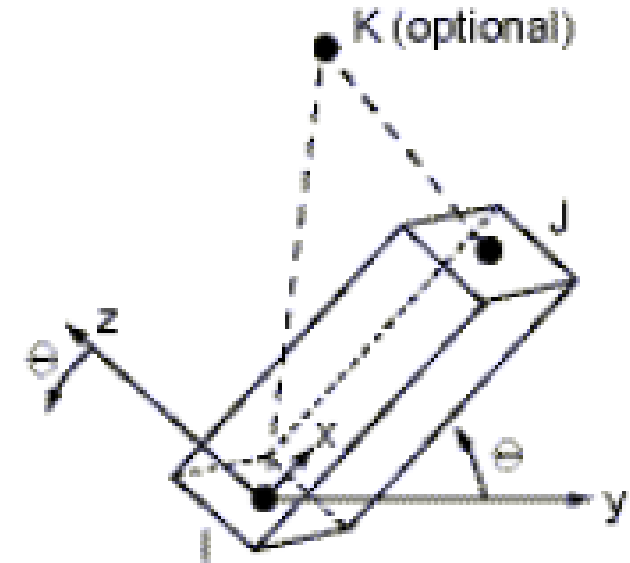


ELEMENTO TRAVE/22

Accesso ai risultati per l'elemento trave 3D – Comando ETABLE

Table 4.3 BEAM4 Item and Sequence Numbers (KEYOPT(9) = 0)

Output Quantity Name	ETABLE and ESOL Command Input			
	Item	E	I	J
MFORX	SMISC	-	1	7
MFORY	SMISC	-	2	8
MFORZ	SMISC	-	3	9
MMOMX	SMISC	-	4	10
MMOMY	SMISC	-	5	11
MMOMZ	SMISC	-	6	12



Torsione

ETABLE, MZI, SMISC, 4 ! Nodo I

ETABLE, MZJ, SMISC, 10 ! Nodo J

MFORX = Forza normale

MFORY = Taglio "y"

MFORZ = Taglio "z"

MMOMX = Torsione

MMOMY = Flessione "y"

MMOMZ = Flessione "z"

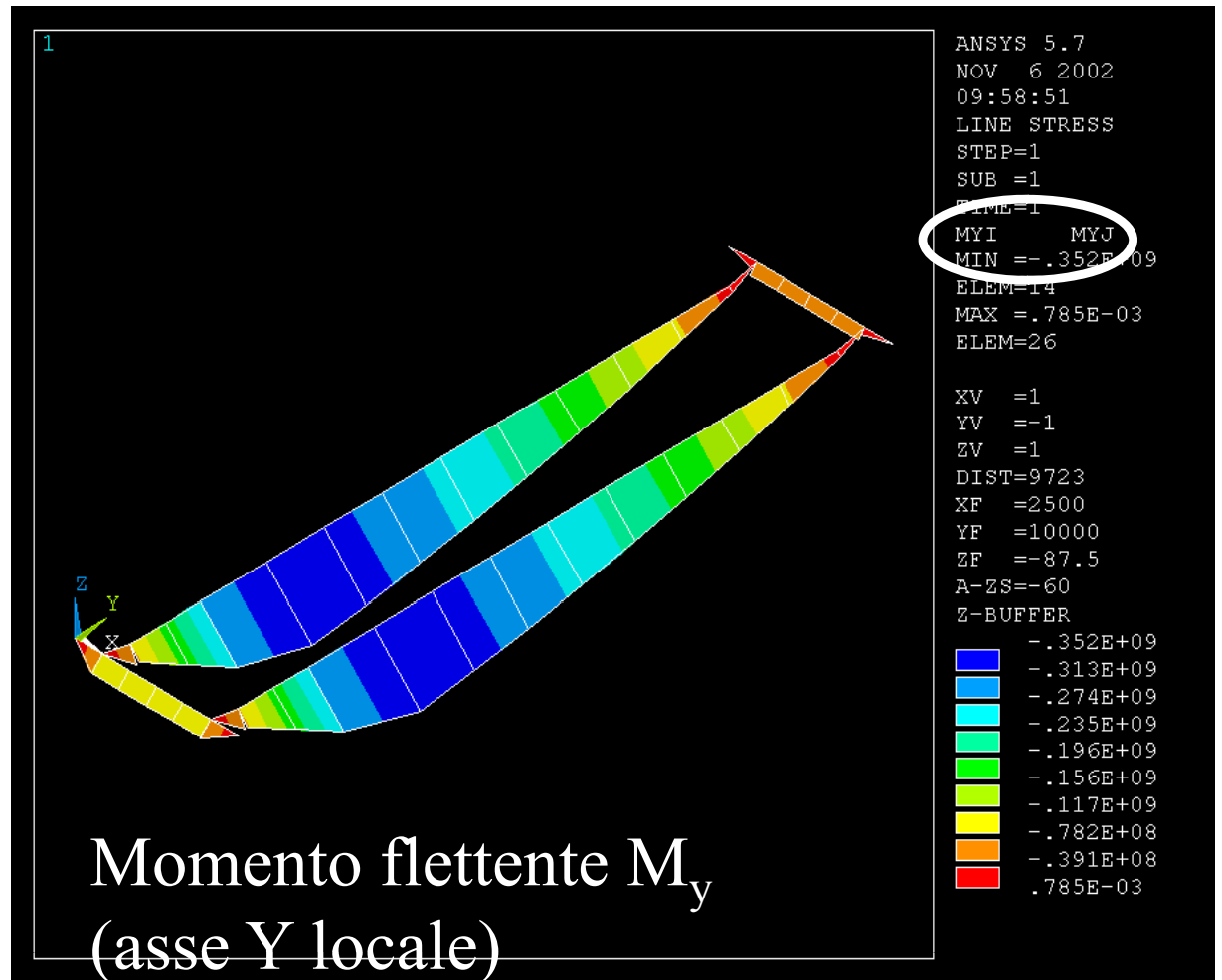
ELEMENTO TRAVE/23

Rappresentazione grafica risultati – Comando PLLS

PLLS, LabI, LabJ, Fact, KUND

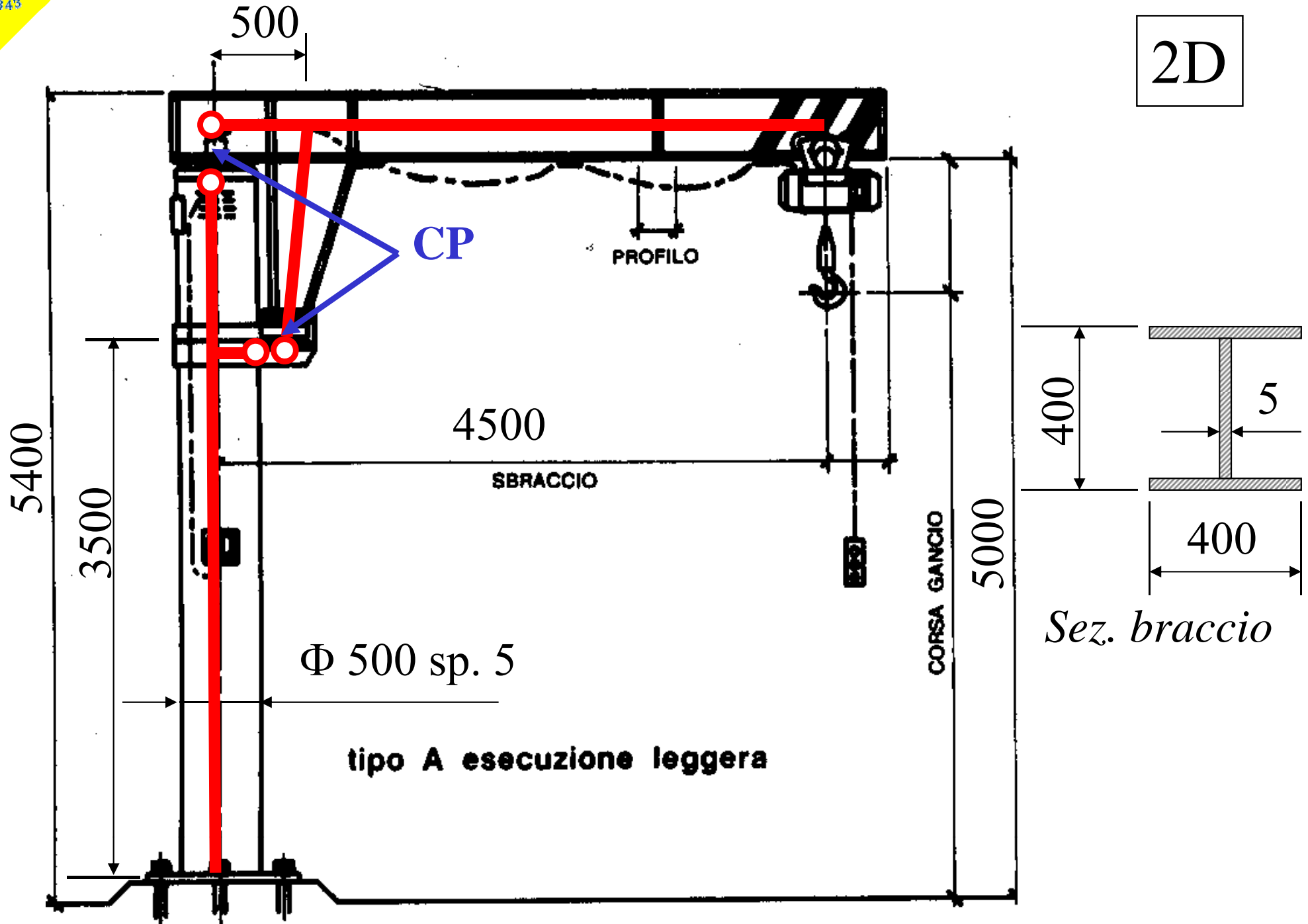
ETABLE, MYI, SMISC, 5
 ETABLE, MYJ, SMISC, 11

PLLS, MYI, MYJ



ELEMENTO TRAVE/24 - GRU A BANDIERA

2D

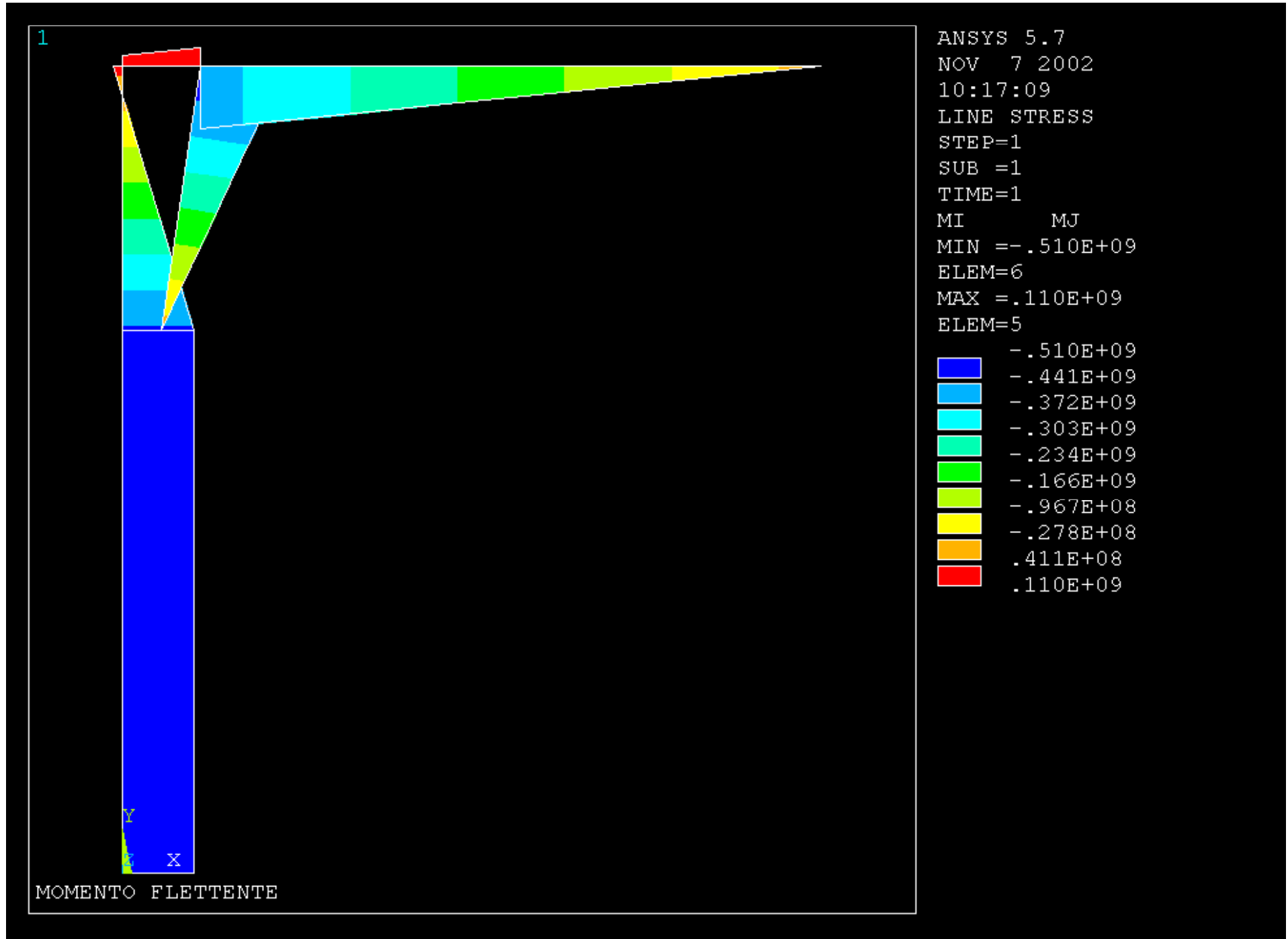




ELEMENTO TRAVE/25 – GRU A BANDIERA

Scuola di Dottorato in Ingegneria "Leonardo da Vinci"

File di comandi: GRU_A_BANDIERA.txt





ASPETTI PARTICOLARI DEL MODELLO

C***

C*** VINCOLI

C***

D,1,ALL,0 ! incastro base colonna

CP,1,UX,3,7 ! appoggio orizzontale inferiore colonna-braccio

CP,2,UX,4,5 ! cerniera superiore colonna-braccio

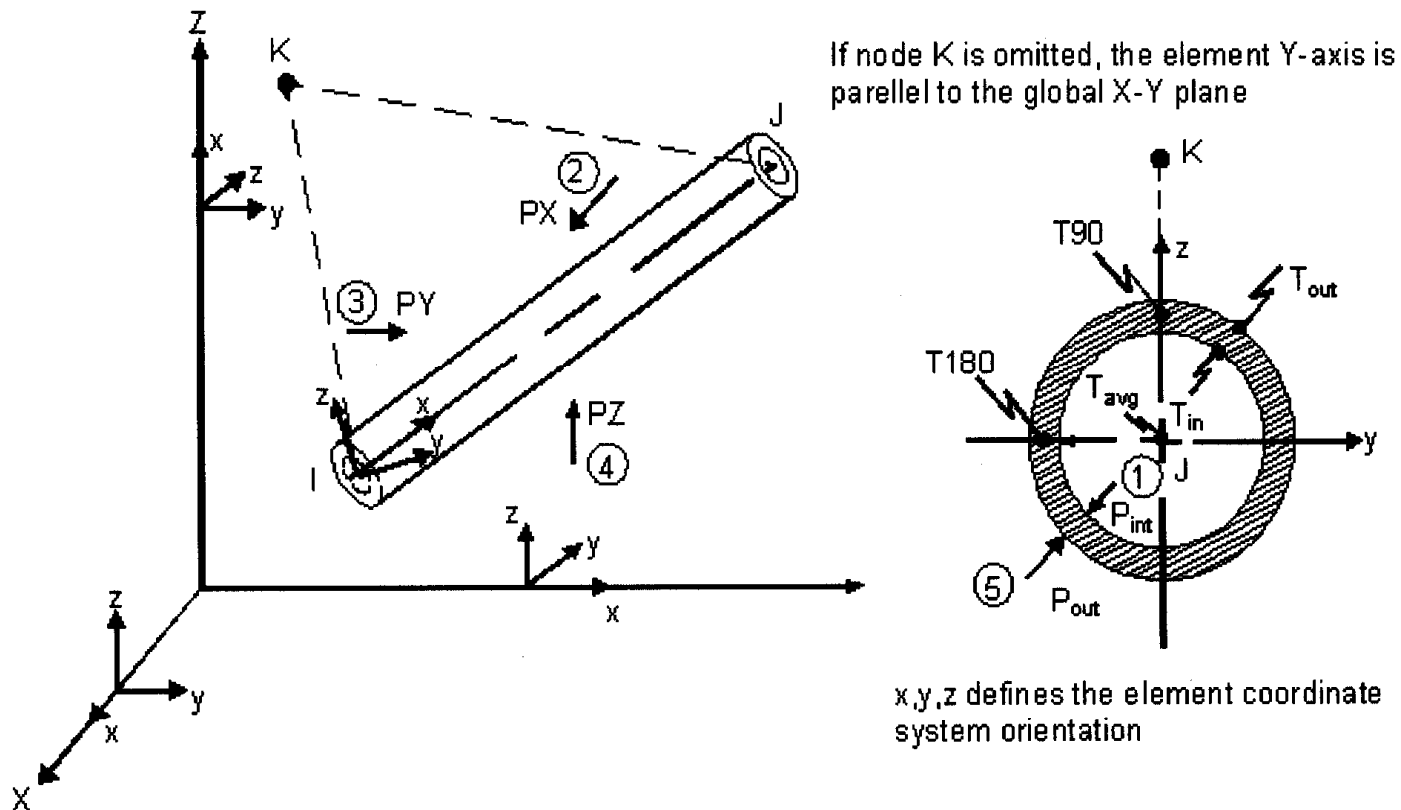
CP,3,UY,4,5

ELEMENTI “PIPE”/1

Serie di elementi per lo studio di sistemi di tubazioni (“piping”) in 2 o 3 dimensioni

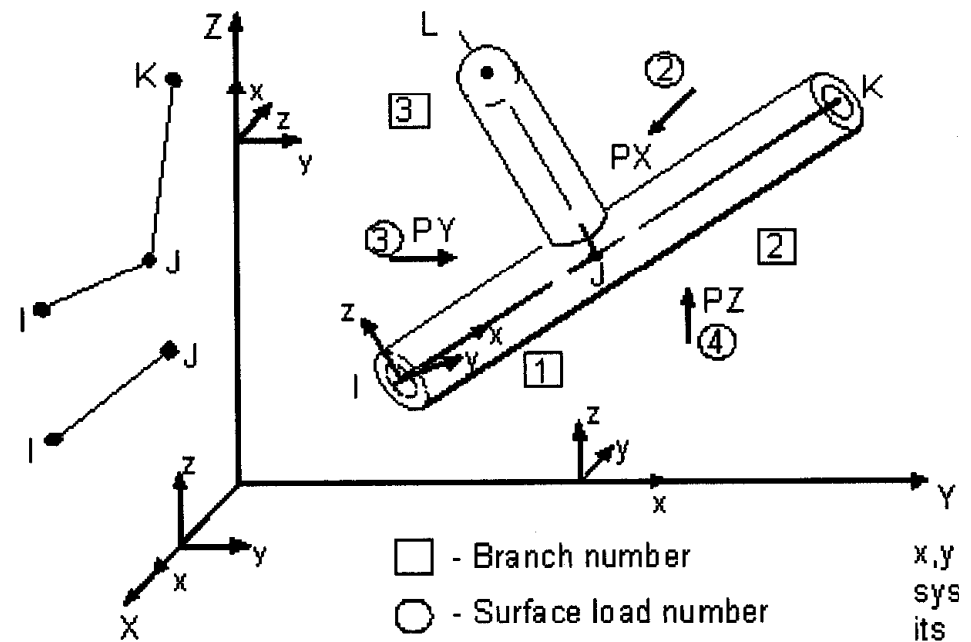
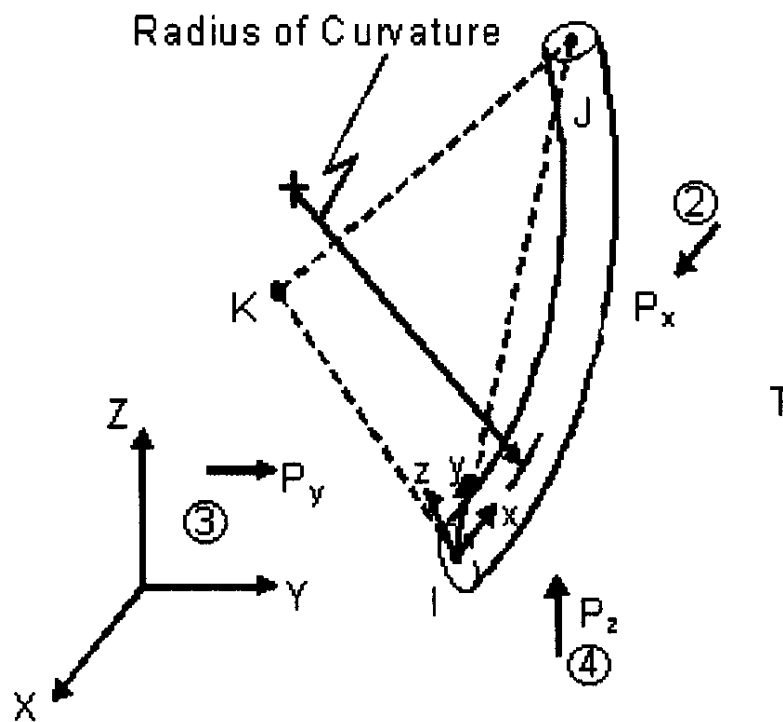
- *tubo rettilineo: elemento trave con un’apposita definizione dei parametri geometrici (diametri invece di A , J , etc.)*

Figure 1. PIPE16 Elastic Straight Pipe



ELEMENTI “PIPE”/2

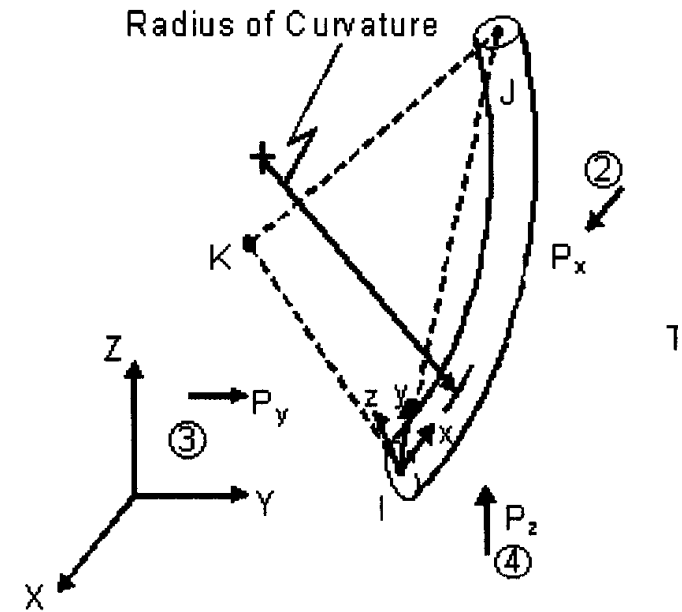
- *tubo curvilineo*: elemento con una speciale definizione della matrice di rigidezza, che tiene conto del basso rapporto tra raggio di curvatura e diametro
- *elementi speciali*: finalizzati a rappresentare correttamente la rigidezza di molti componenti tipici (“T”, valvole, etc.)



ELEMENTI “PIPE”/3

Elemento “tubo curvo” - PARAMETRI RILEVANTI PER FLESSIBILITA’

r = average radius
 R = radius of curvature
 E = modulus of elasticity
 t = thickness



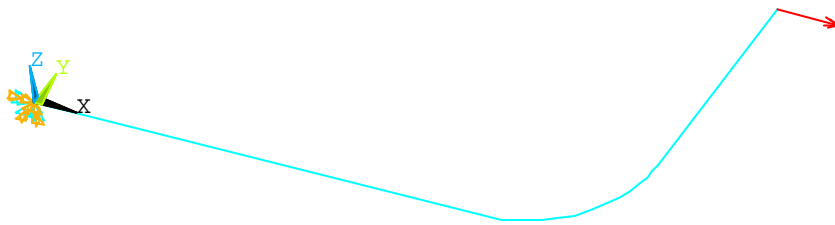
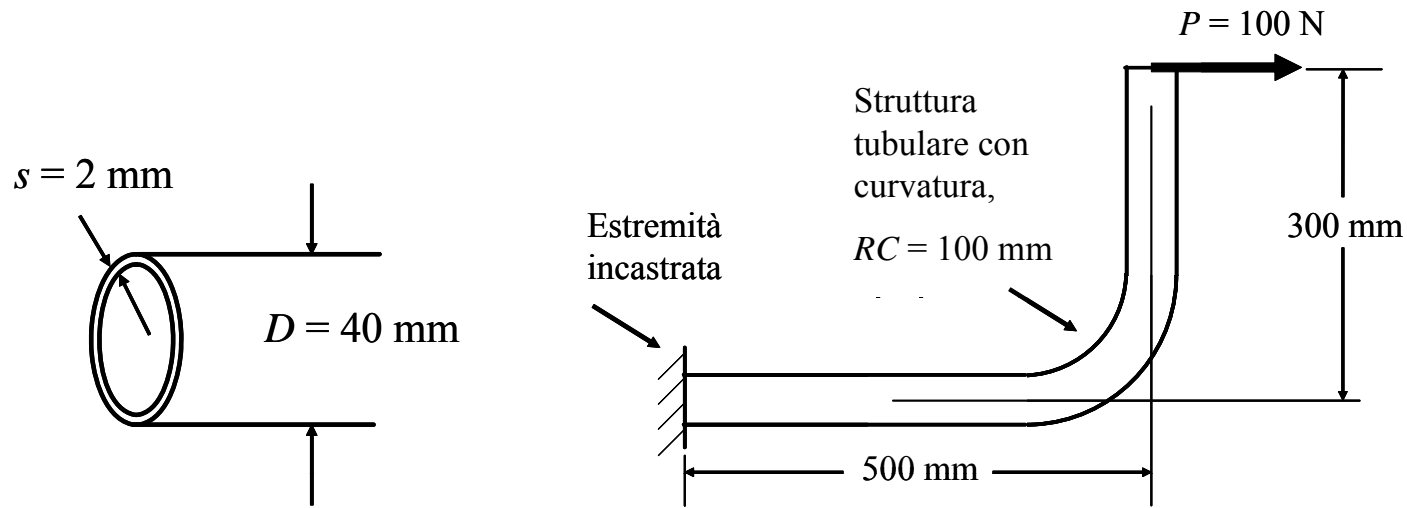
- Fattori di variazione della flessibilità (Flexibility factors: relazioni semi-empiriche di “best-fit”):
 - ANSYS Flexibility Factor = $1.65/(h(1 + PrX_k/tE))$ or 1.0 (whichever is greater)
 - Karman Flexibility Factor = $(10 + 12h^2)/(1 + 12h^2)$
- Fattore di intensificazione delle tensioni = $0.9/h^{2/3}$ or 1.0 (whichever is greater)

$$h = tR/r^2$$

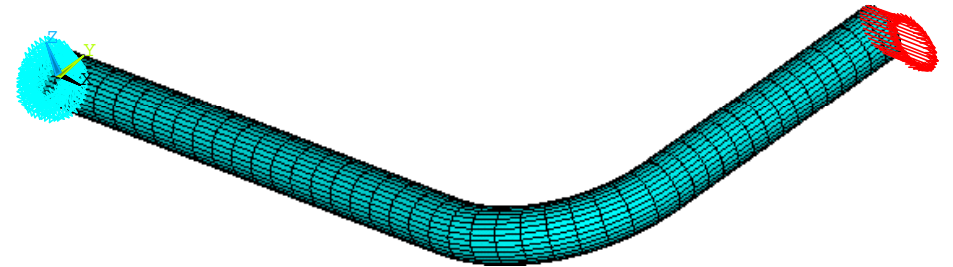
$$P = P_i - P_o \text{ if } P_i - P_o > 0, \text{ otherwise } P = 0, P_i = \text{internal pressure}, P_o = \text{external pressure}$$

$$X_k = 6 (r/t)^{4/3} (R/r)^{1/3} \text{ if KEYOPT}(3) = 1 \text{ and } R/r > 1.7, \text{ otherwise } X_k = 0$$

ELEMENTI “PIPE”/3



Modello con elementi “Pipe” e “Beam”



Modello 3D completo (shell)



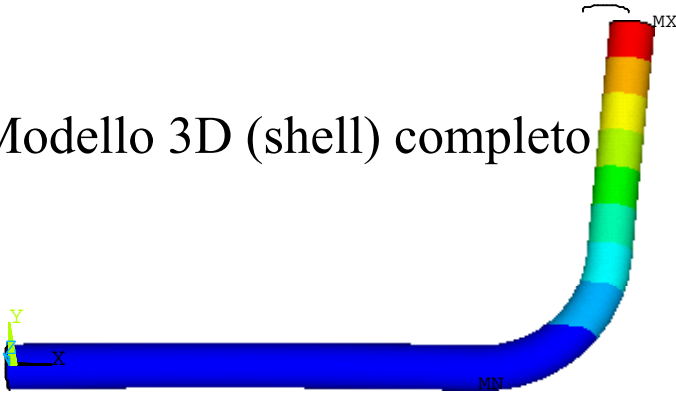
ELEMENTI “PIPE”/4

Risultato “esatto” (Modello 3D completo)

```

TIME=1
UX      (AVG)
RSYS=0
DMX = .899508
SMN = -.025122
SMX = .748652
  
```

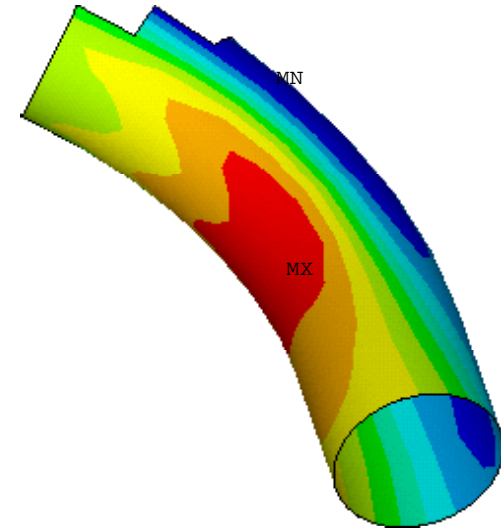
U_x – Modello 3D (shell) completo



```

TIME=1
SX      (AVG)
RSYS=0
DMX = .419267
SMN = -17.964
SMX = 20.739
  
```

Tensione





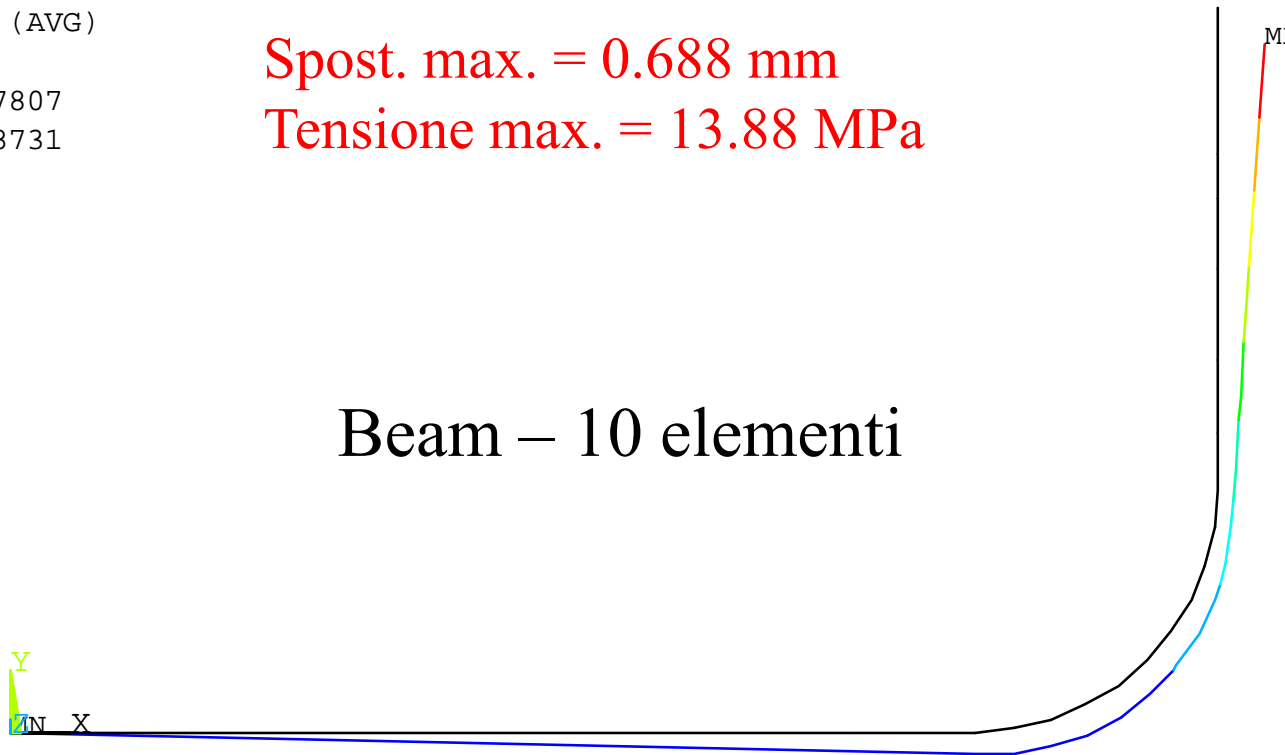
ELEMENTI “PIPE”/4

Risultato “esatto”: Spost. max.= 0.8995 mm
 Tensione max. = 20.74 MPa

TIME=1
UX (AVG)
RSYS=0
DMX = .687807
SMX = .548731

Spost. max. = 0.688 mm
Tensione max. = 13.88 MPa

Beam – 10 elementi





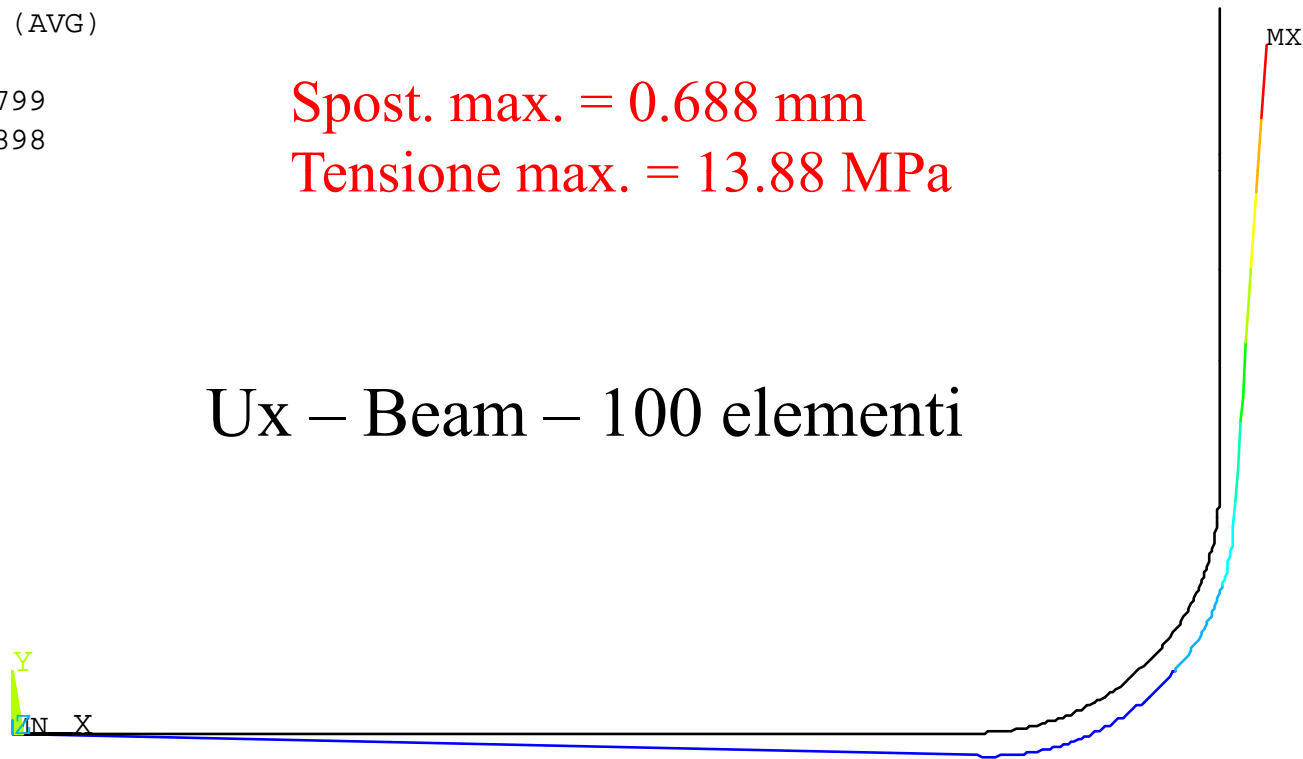
ELEMENTI “PIPE”/4

Risultato “esatto”: Spost. max.= 0.8995 mm
 Tensione max. = 20.74 MPa

TIME=1
UX (AVG)
RSYS=0
DMX = .68799
SMX = .54898

Spost. max. = 0.688 mm
Tensione max. = 13.88 MPa

Ux – Beam – 100 elementi





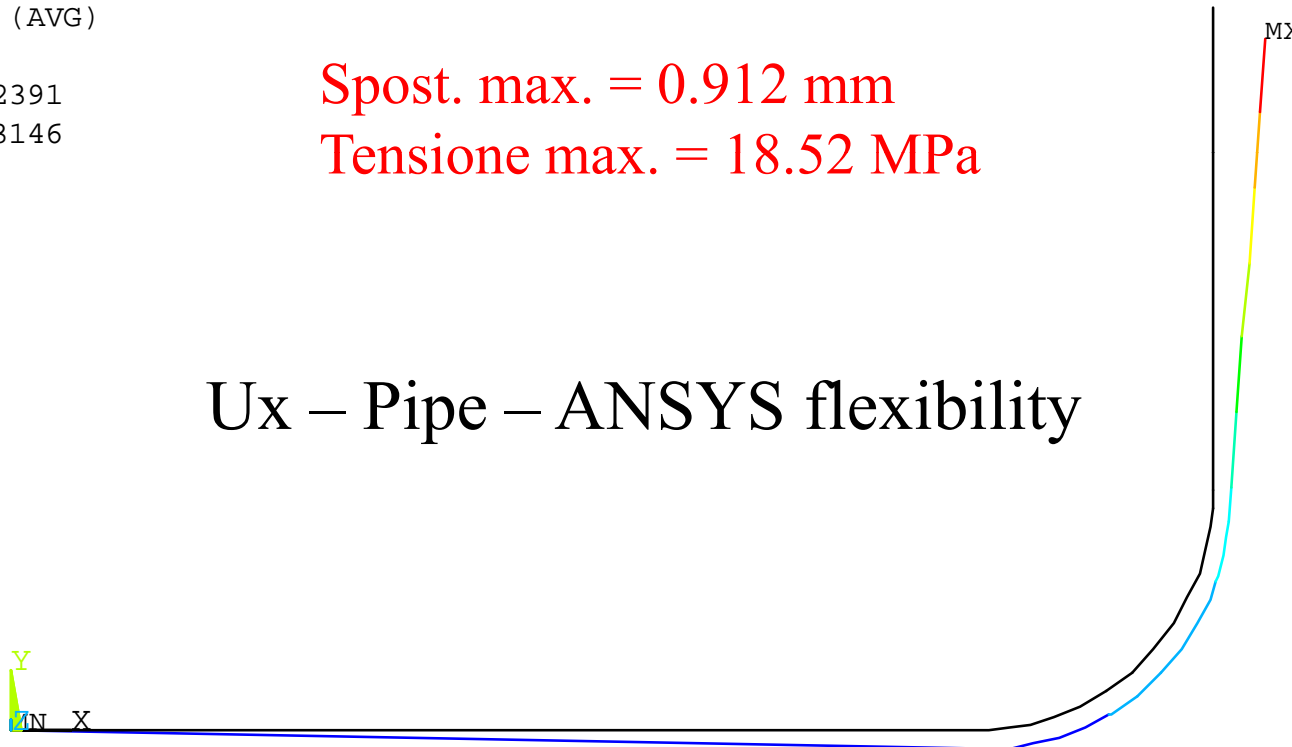
ELEMENTI “PIPE”/4

Risultato “esatto”: Spost. max.= 0.8995 mm
 Tensione max. = 20.74 MPa

```
TIME=1  
UX        (AVG)  
RSYS=0  
DMX = .912391  
SMX = .793146
```

Spost. max. = 0.912 mm
Tensione max. = 18.52 MPa

U_x – Pipe – ANSYS flexibility





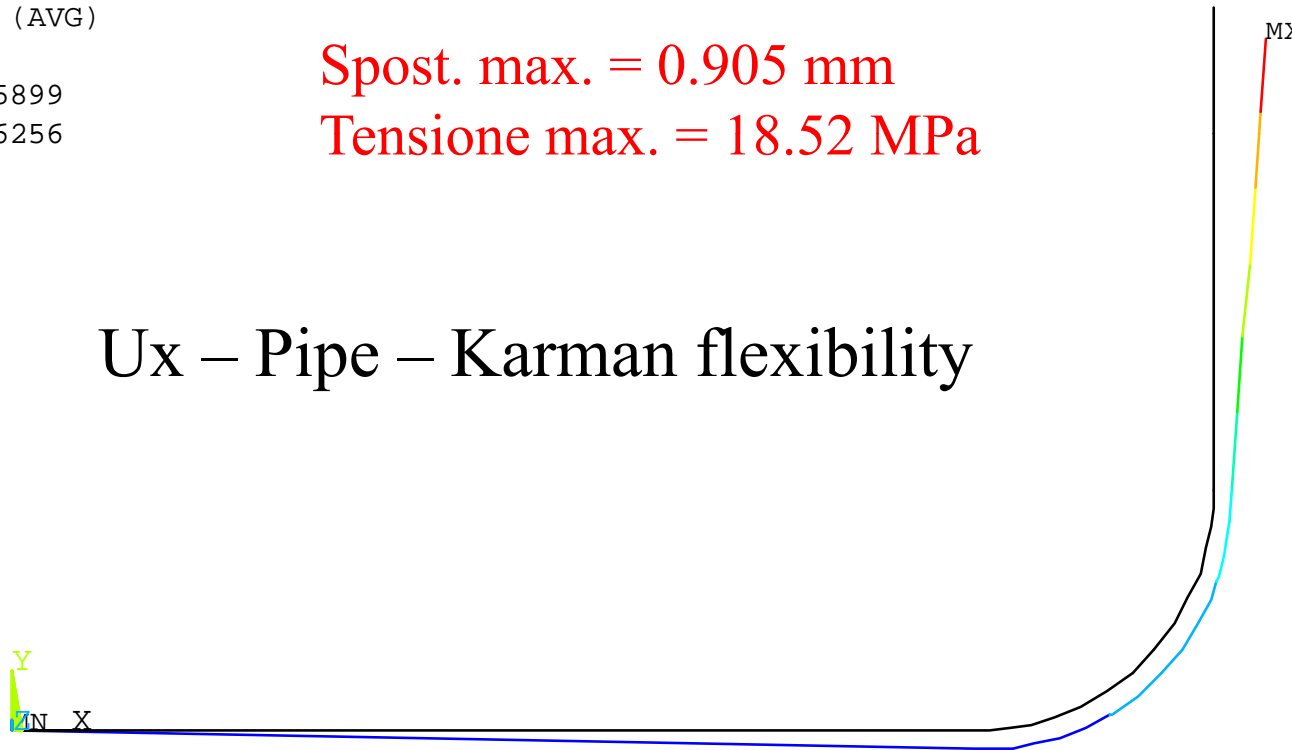
ELEMENTI “PIPE”/4

Risultato “esatto”: Spost. max.= 0.8995 mm
 Tensione max. = 20.74 MPa

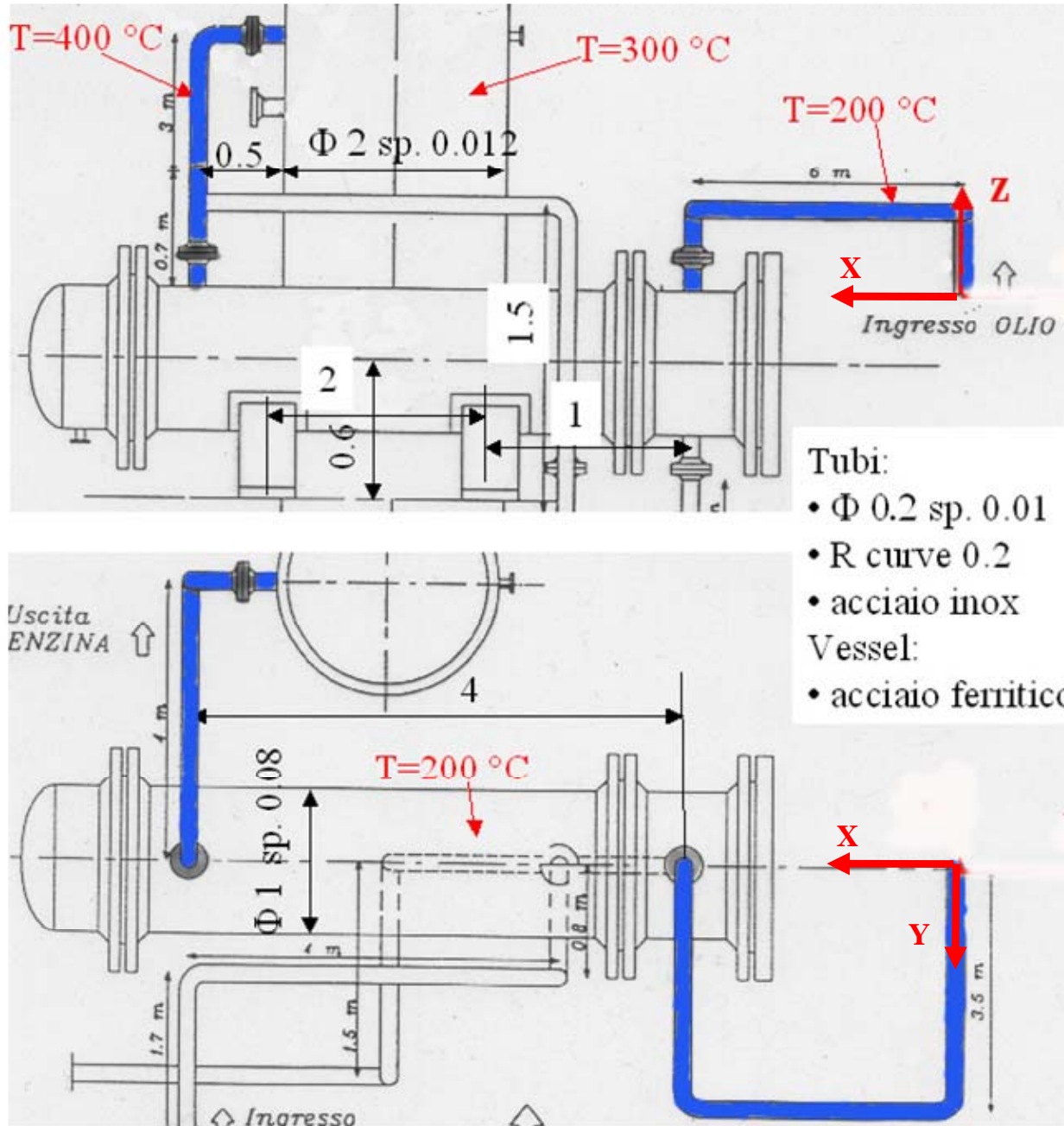
```
--- -  
TIME=1  
UX            (AVG)  
RSYS=0  
DMX = .905899  
SMX = .786256
```

Spost. max. = 0.905 mm
Tensione max. = 18.52 MPa

Ux – Pipe – Karman flexibility



ELEMENTI "PIPE"/3

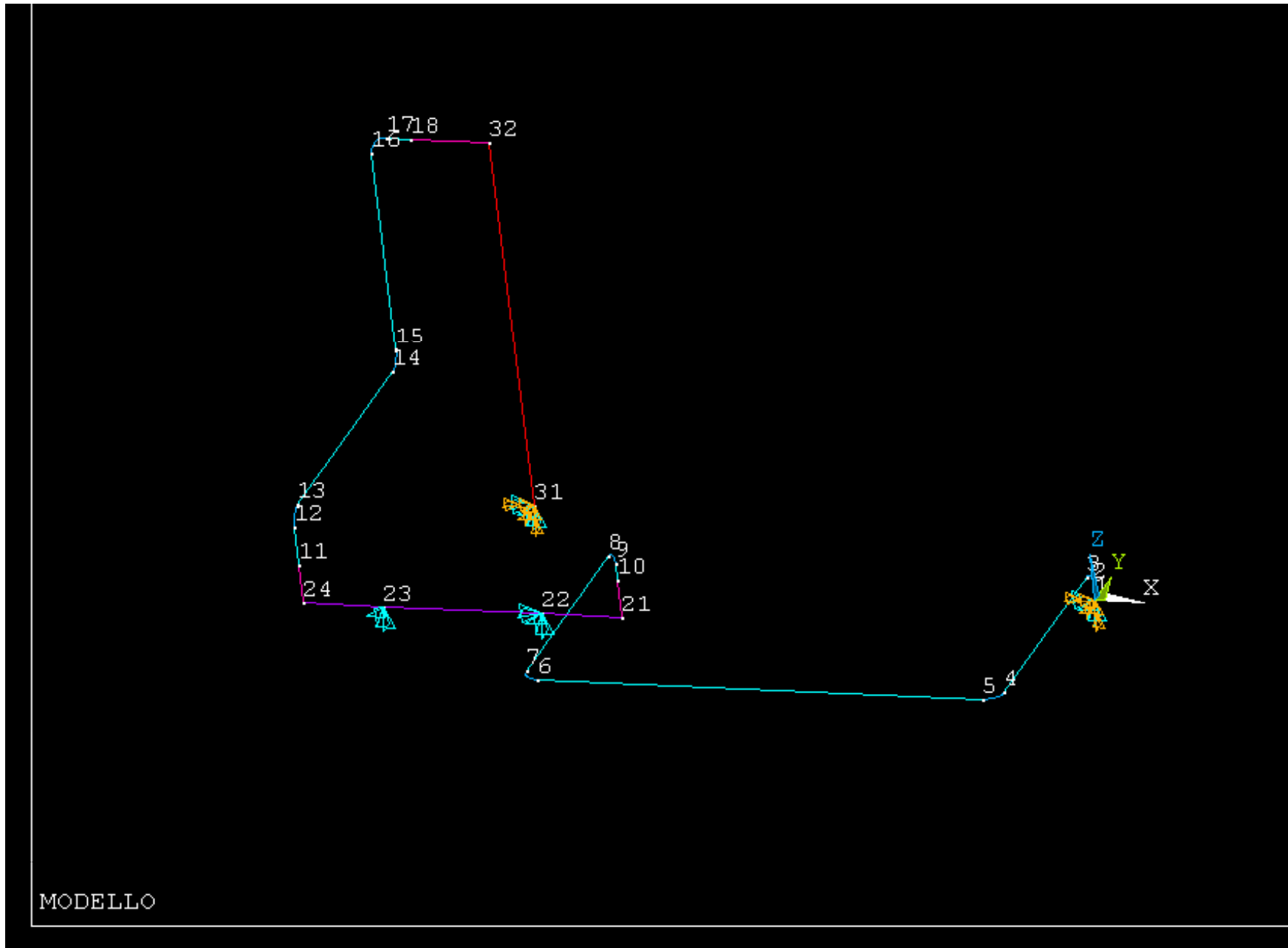


*Dimensioni
espresse in
metri*

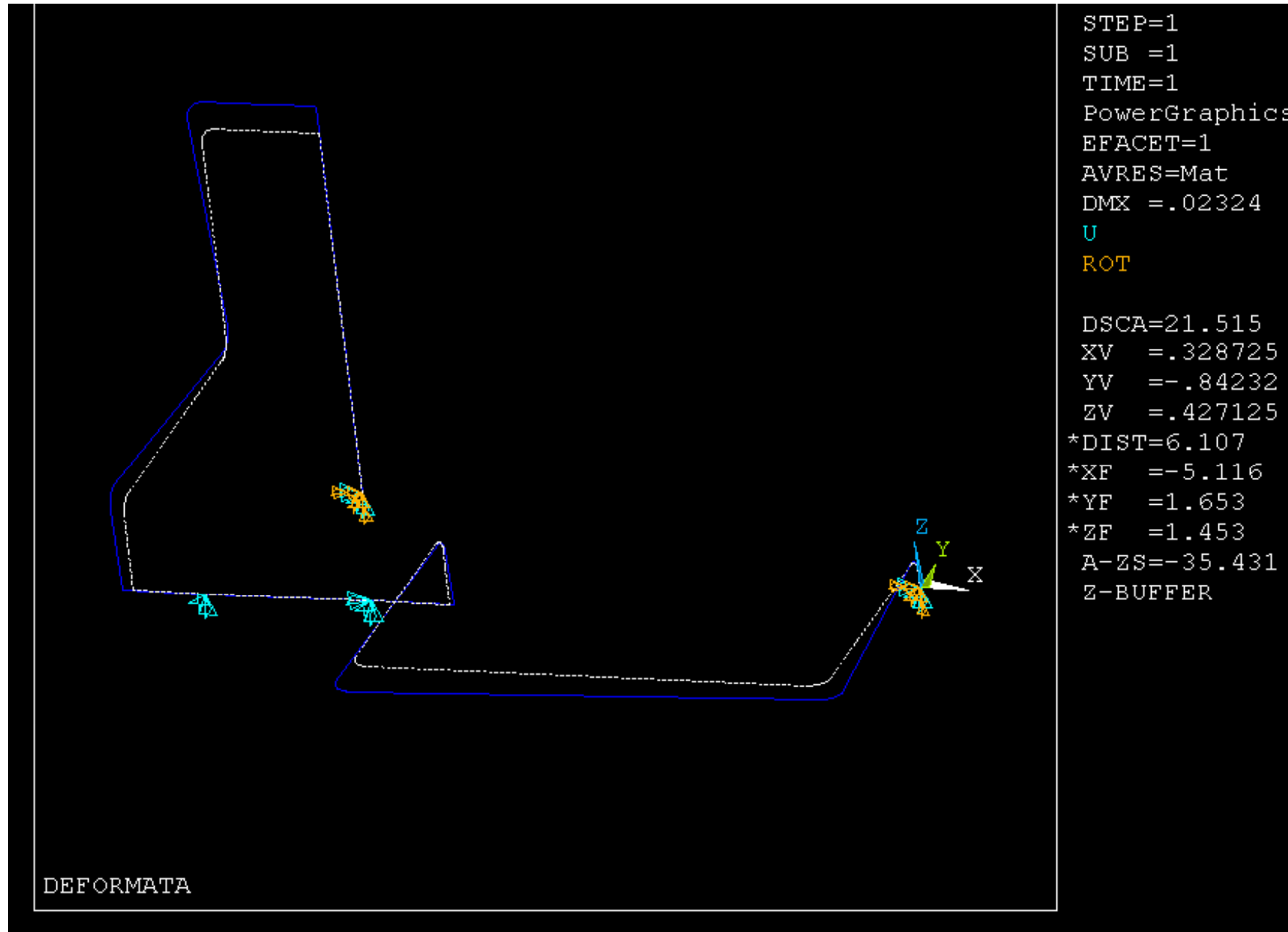
Il modello
rappresenta i
tratti di
tubazione di
colore blu ed i
2 vessel

*File comandi:
"piping.txt"*

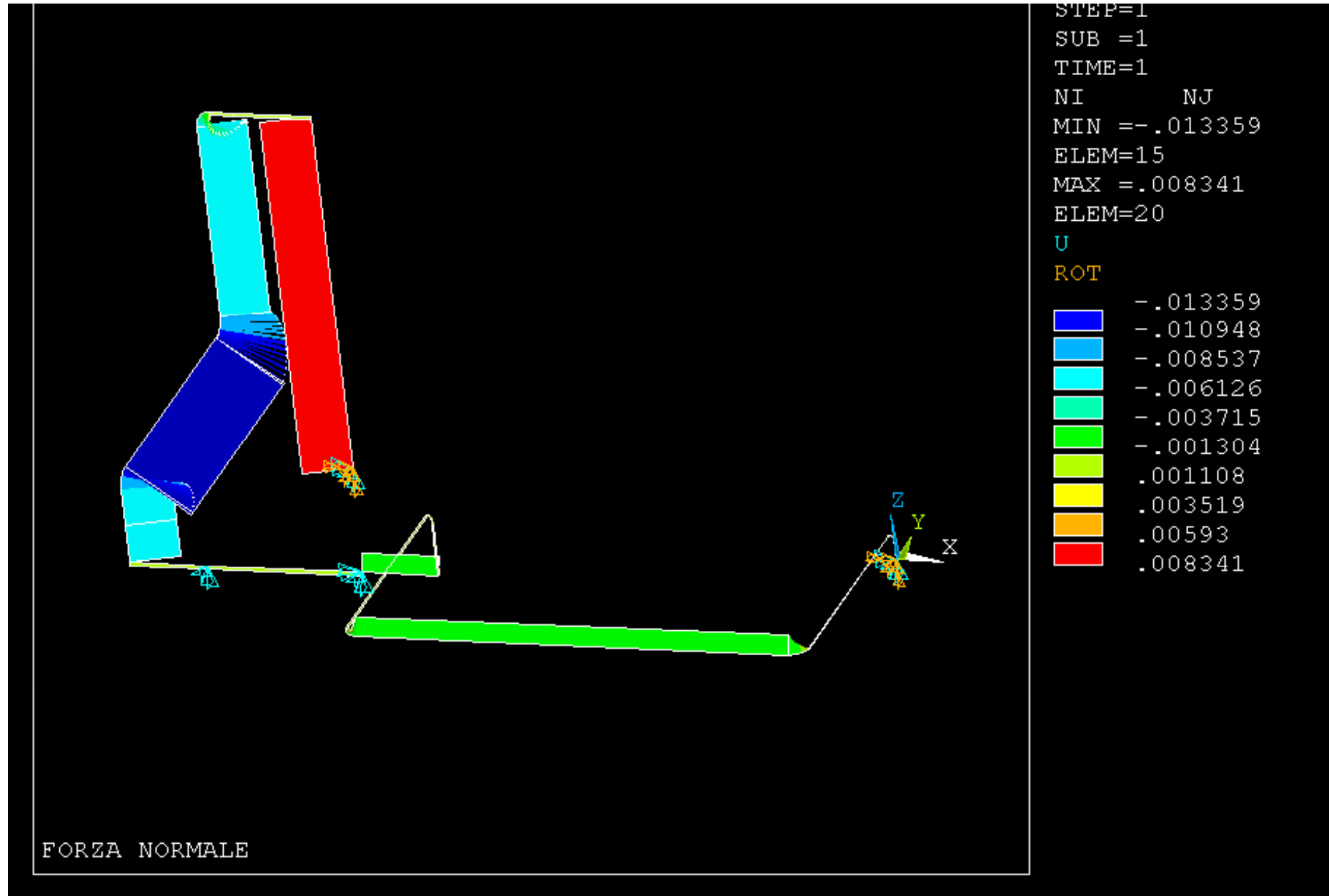
ELEMENTI “PIPE”/4



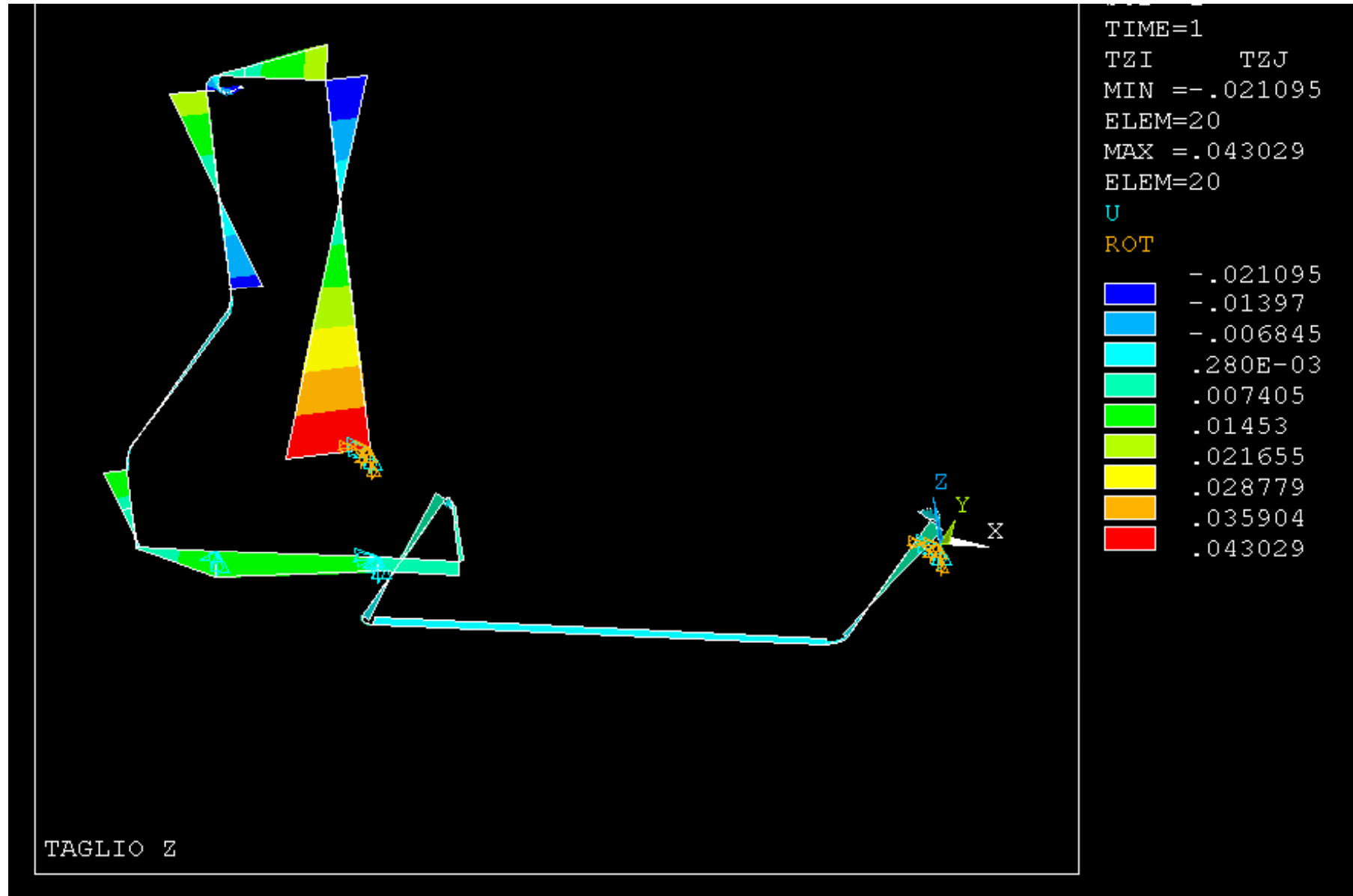
ELEMENTI “PIPE”/4



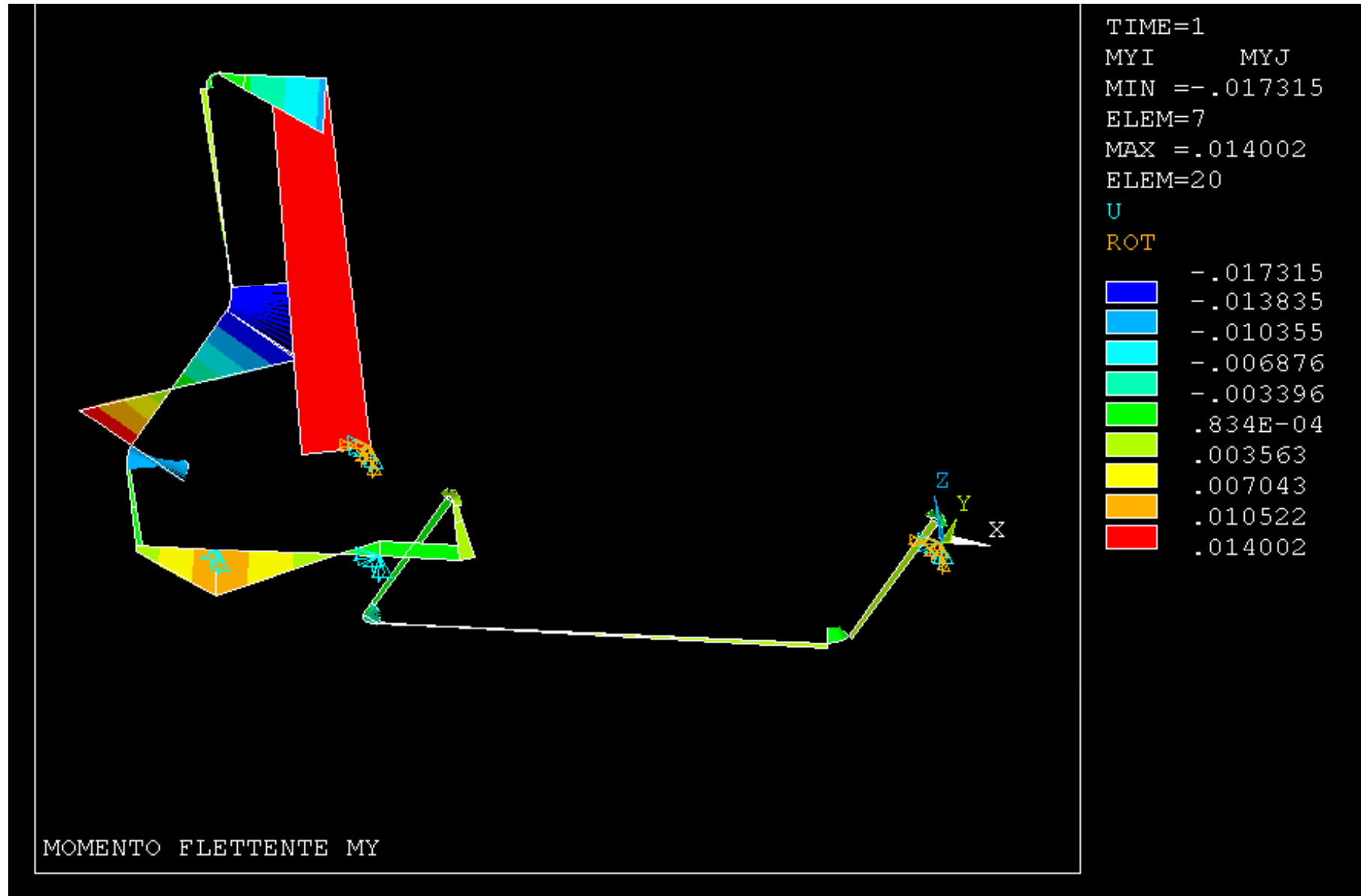
ELEMENTI “PIPE”/4



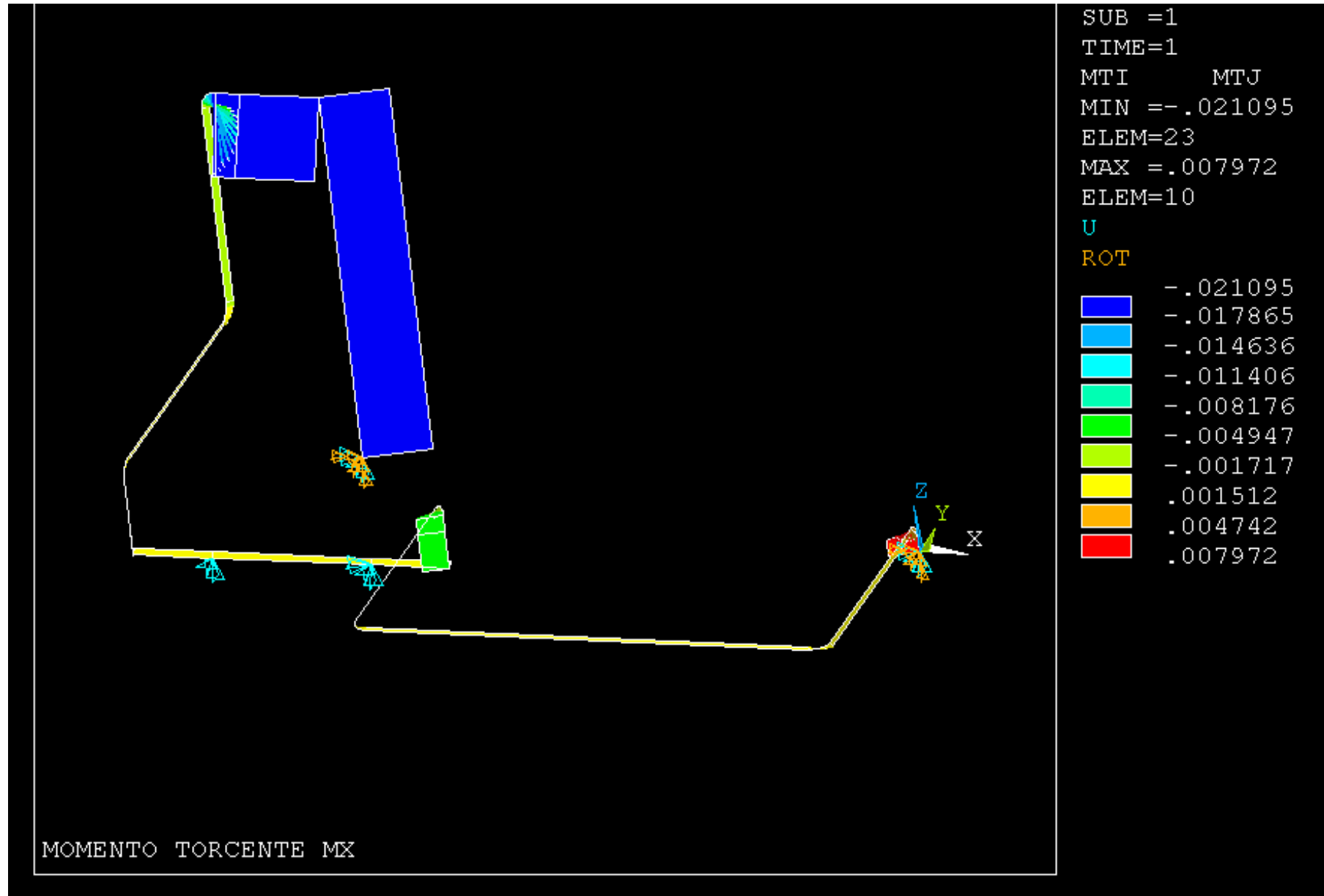
ELEMENTI “PIPE”/4



ELEMENTI “PIPE”/4



ELEMENTI “PIPE”/4





ASPETTI PARTICOLARI DEL MODELLO

COEFFICIENTE DI DILATAZIONE TERMICA

C***

C*** MATERIALE

C***

C*** acciaio inox

C***

MP,EX,1,200000

MP,ALPX,1,0.000018 ! coefficiente di dilatazione termica

C***

C*** acciaio ferritico

C***

MP,EX,2,210000

MP,ALPX,2,0.000012 ! coefficiente di dilatazione termica



ASPETTI PARTICOLARI DEL MODELLO

CARICHI DOVUTI A VARIAZIONI DI TEMPERATURA

C***

C*** CARICHI

C***

C*** tubazione olio

C***

TREF,20

! temperatura di montaggio

NSEL,,NODE,,1,10

BF,ALL,TEMP,200

! temperatura di lavoro

C***

C*** tubazione benzina

C***

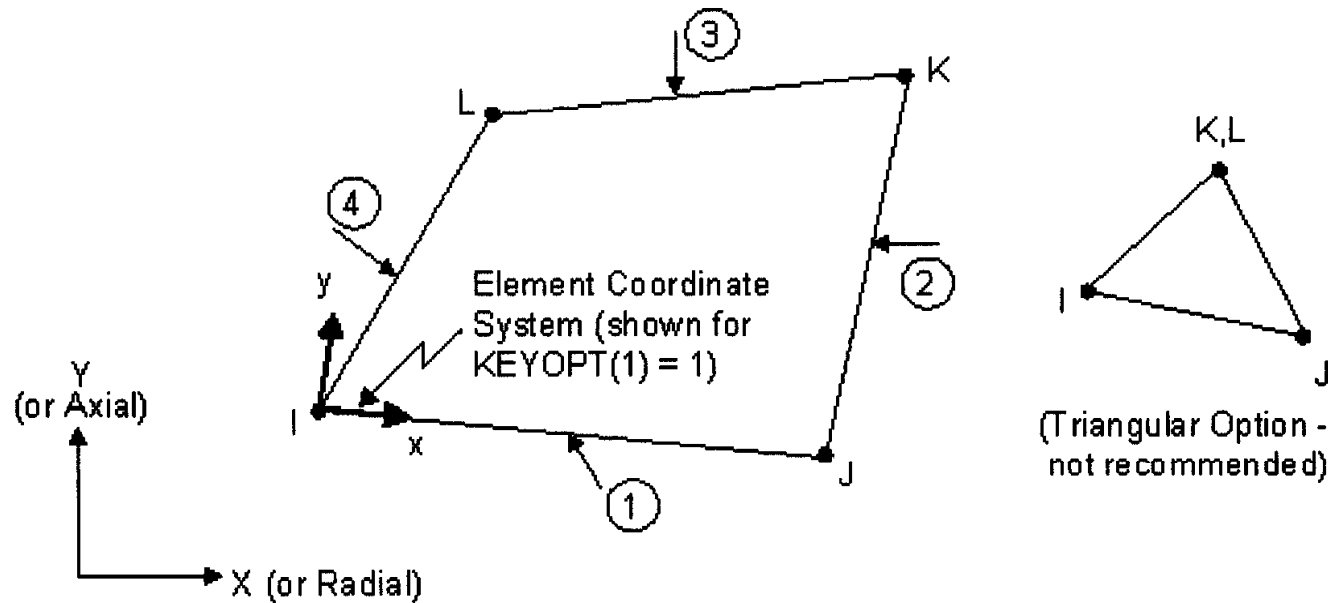
NSEL,,NODE,,11,18

BF,ALL,TEMP,400

! temperatura di lavoro

ELEMENTI PIANI/1

Figure 1. PLANE42 2-D Structural Solid

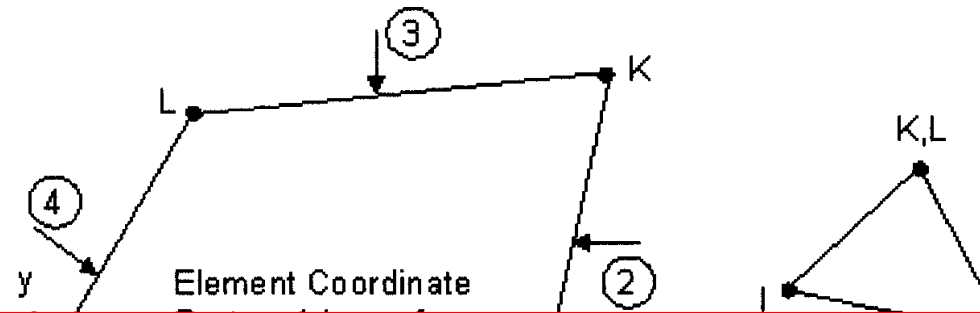


Problemi di elasticità piana

- 4 (3) nodi
- 2 g.d.l /nodo
- tre “classi” di problemi:
 - *stati piani di tensione (“plane stress”)*
 - *stati piani di deformazione (“plane strain”)*
 - *stati assialsimmetrici (“axi-symmetric stress/strain”)*

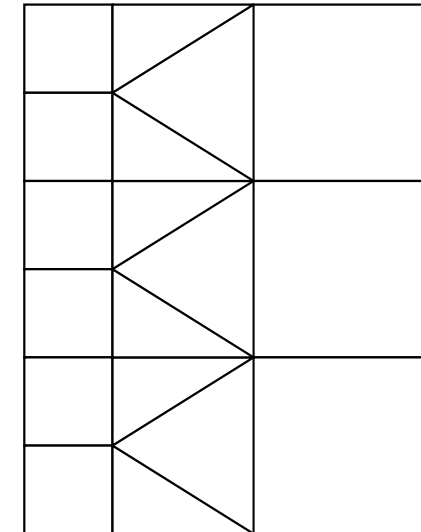
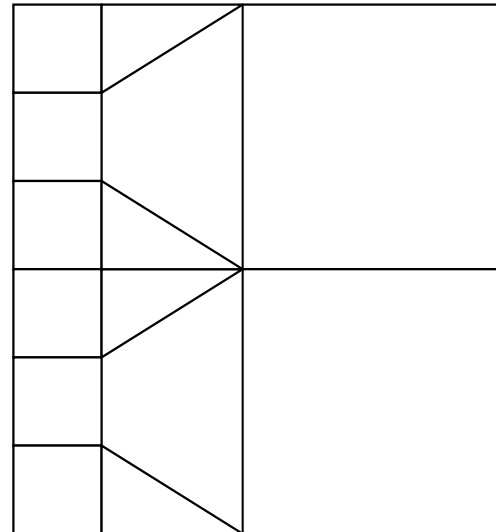
ELEMENTI PIANI/1

Figure 1. PLANE42 2-D Structural Solid



Y
(or Ax)

Esempi di zone di transizione

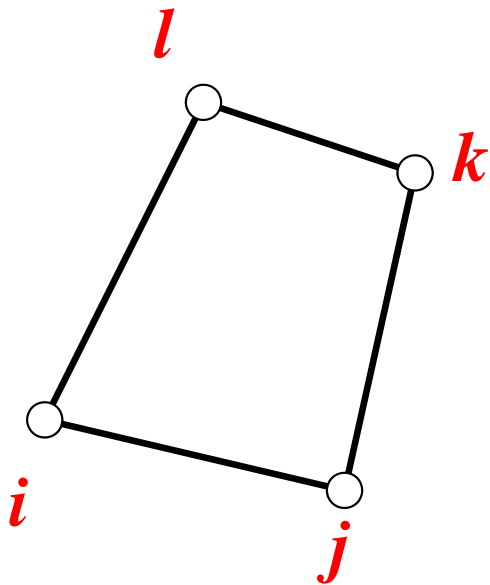


Problemi di elasto

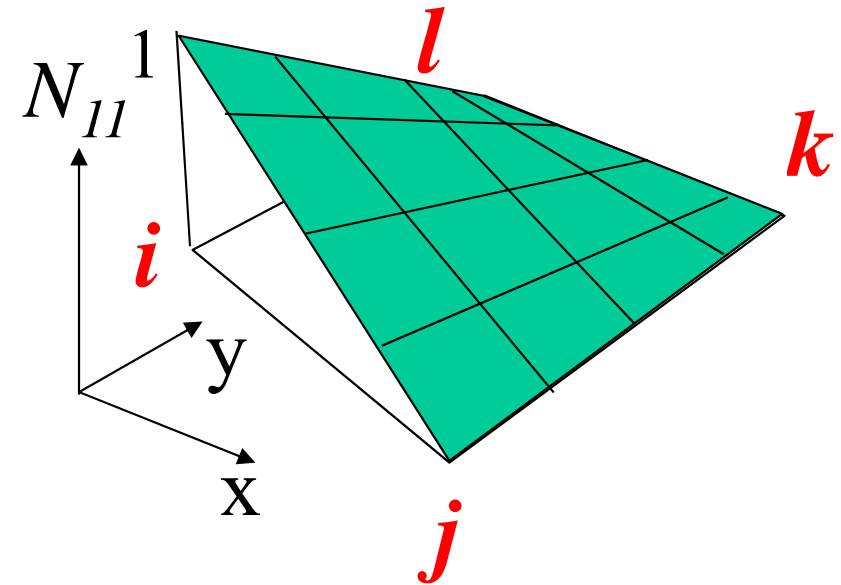
- 4 (3) nodi
- 2 g.d.l /nodo
- tre “classi” di
 - stati piani
 - stati piani di deformazione (“plane strain”)
 - stati assialsimmetrici (“axi-symmetric stress/strain”)

ELEMENTI PIANI/2

Rispetto all'elemento triangolare è possibile scrivere 4 condizioni (invece di 3) per ciascuna delle f.ni di forma



$$\begin{cases} N_{11}(x_i, y_i) = 1 \\ N_{11}(x_j, y_j) = 0 \\ N_{11}(x_k, y_k) = 0 \\ N_{11}(x_l, y_l) = 0 \end{cases}$$



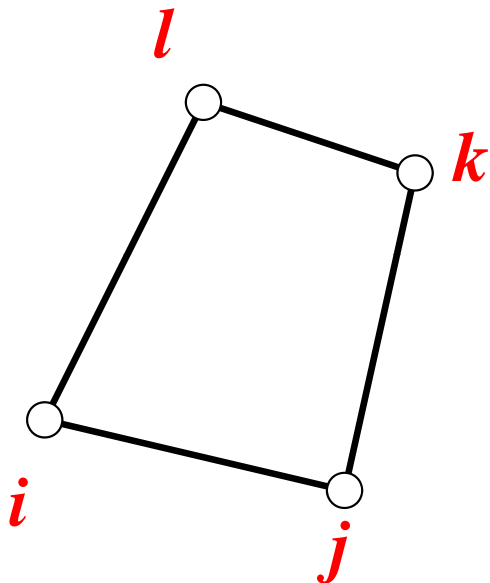
Per tale motivo, le f.ni di forma possono avere una formulazione a 4 parametri, che include un termine di 2° grado

$$N_{11}(x, y) = A_{11} + B_{11}x + C_{11}y + D_{11}xy$$

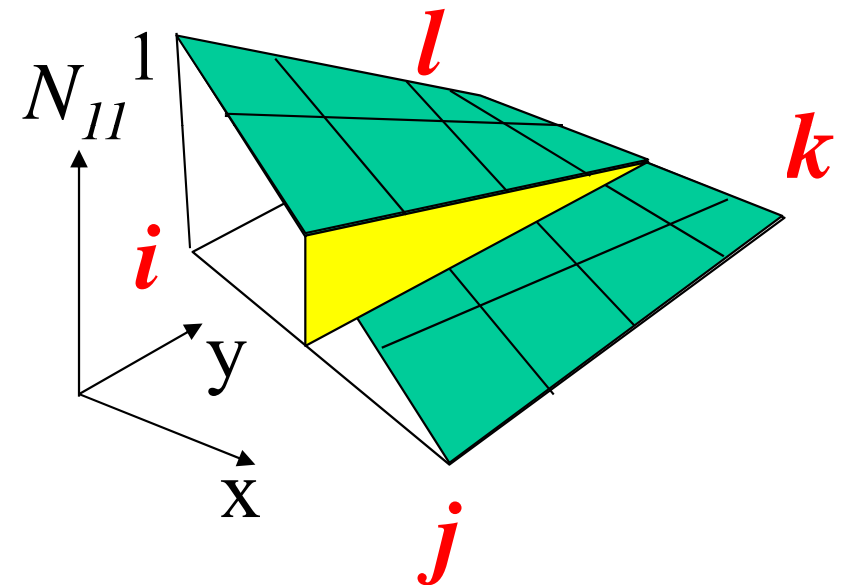
ELEMENTI PIANI/2

Rispetto all'elemento triangolare è più complesso (invece di 3) per ciascuna delle f.ni di

Superficie rigata: ogni sezione con piani “ $x=cost$ ” mostra una variazione lineare con “ y ” e viceversa



$$\begin{cases} N_{11}(x_i, y_i) = 1 \\ N_{11}(x_j, y_j) = 0 \\ N_{11}(x_k, y_k) = 0 \\ N_{11}(x_l, y_l) = 0 \end{cases}$$



Per tale motivo, le f.ni di forma possono avere una formulazione a 4 parametri, che include un termine di 2° grado

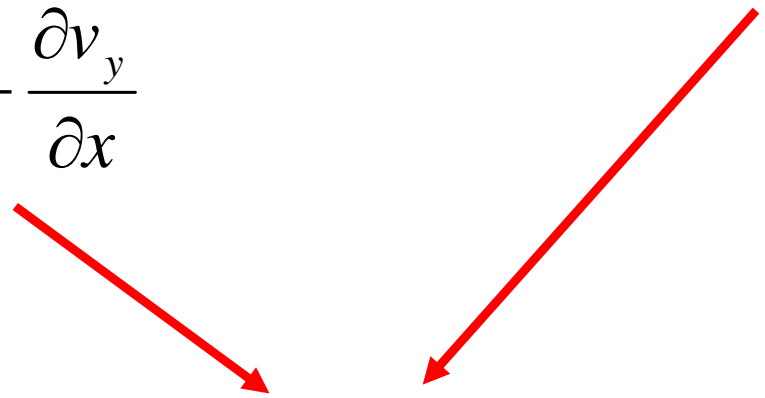
$$N_{11}(x, y) = A_{11} + B_{11}x + C_{11}y + D_{11}xy$$

ELEMENTI PIANI/3

Andamento tensioni/deformazioni

$$\left\{ \begin{array}{l} \varepsilon_x = \frac{\partial v_x}{\partial x} \\ \varepsilon_y = \frac{\partial v_y}{\partial y} \\ \gamma_{xy} = \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \end{array} \right.$$

$$N_{lm}(x, y) = A_{lm} + B_{lm}x + C_{lm}y + D_{lm}xy$$

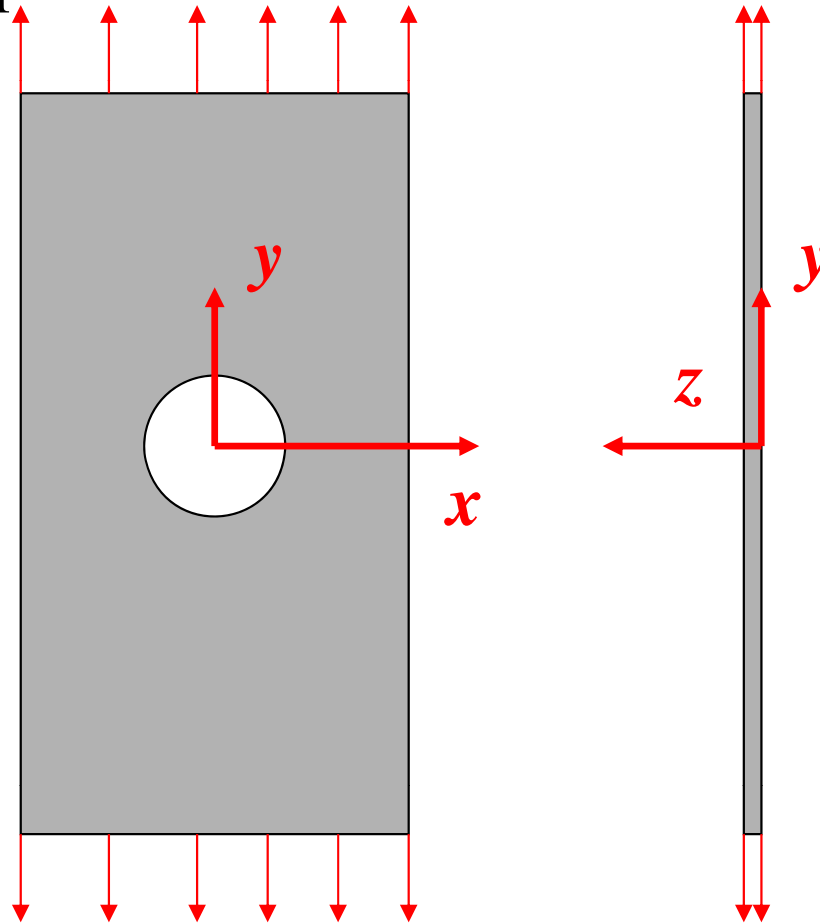


$$\left\{ \begin{array}{l} \varepsilon_x = a + b \cdot y \\ \varepsilon_y = c + d \cdot x \\ \gamma_{xy} = e + f \cdot x + g \cdot y \end{array} \right.$$

ELEMENTI PIANI/4

Stati piani di tensione:

- sono caratterizzati dall'aver una delle componenti principali di tensione identicamente nulla
- si verificano tipicamente in corpi piani, di spessore piccolo rispetto alle altre dimensioni caratteristiche del problema, caricati nel loro piano medio.

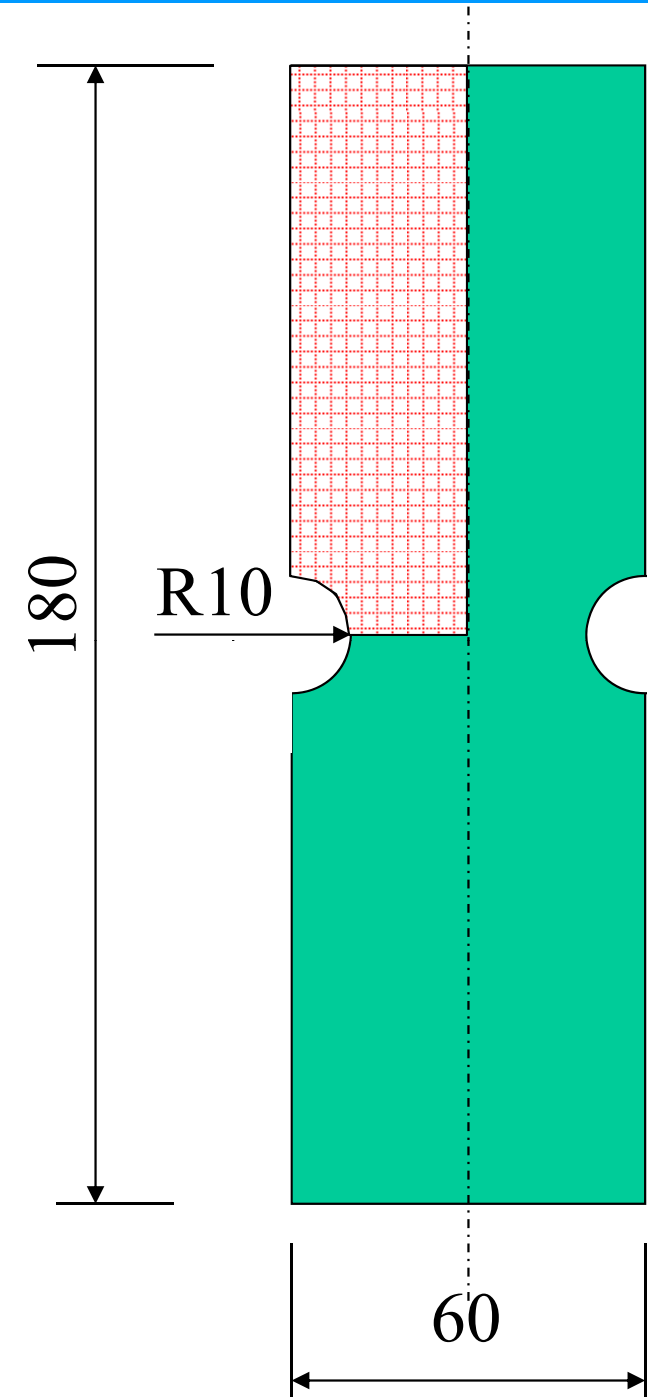
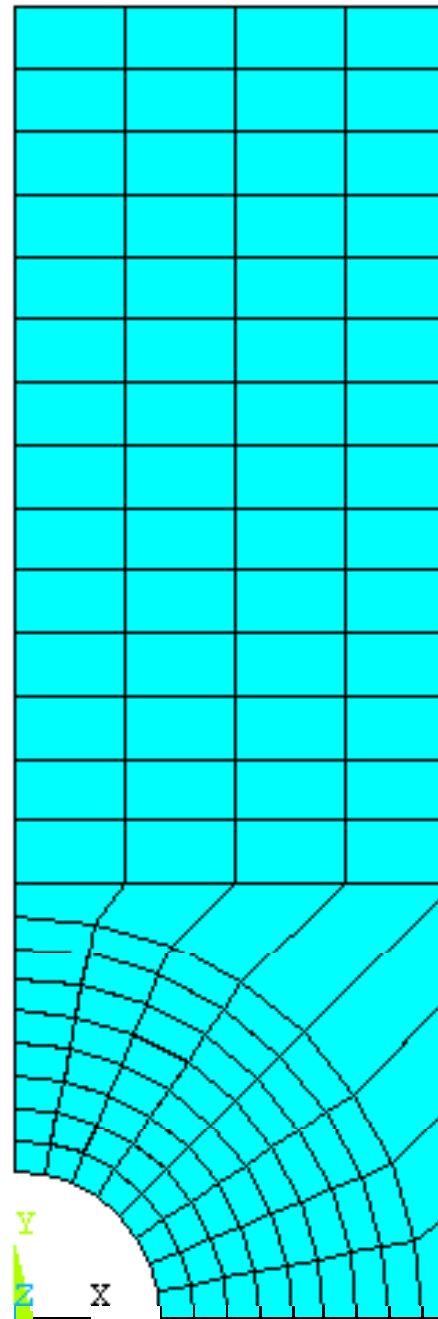


$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

$$\sigma_x, \sigma_y, \tau_{xy} \neq 0$$

ELEMENTI PIANI/5

Il modello giace sul piano “x-y” e rappresenta il piano medio (a metà spessore) della struttura. I carichi possono essere sull’intero spessore o per unità di spessore.



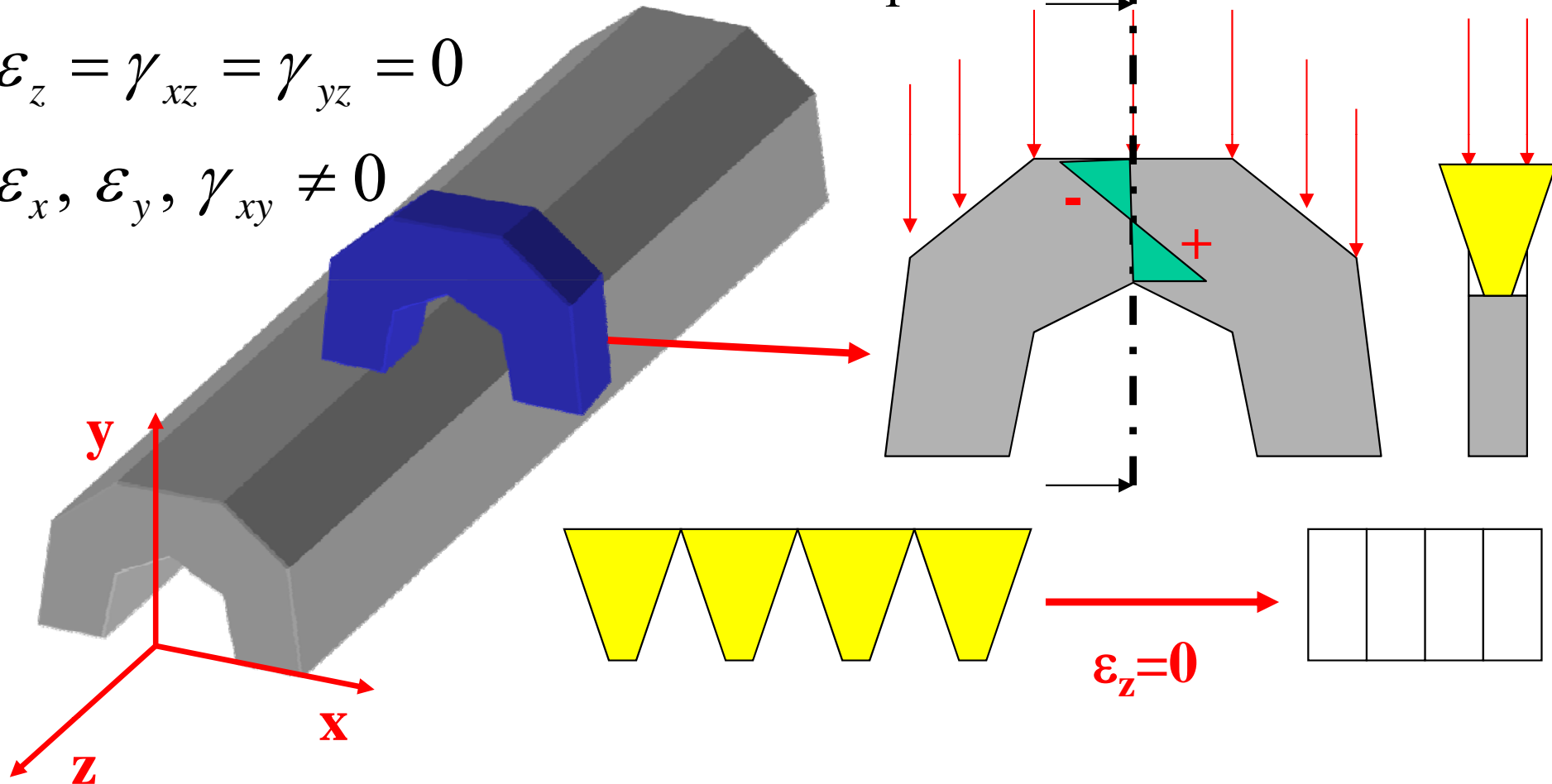
ELEMENTI PIANI/6

Stati piani di deformazione:

- sono caratterizzati dall'avere una delle componenti principali di deformazione identicamente nulla
- si verificano tipicamente in corpi di spessore grande rispetto alle altre dimensioni caratteristiche del problema.

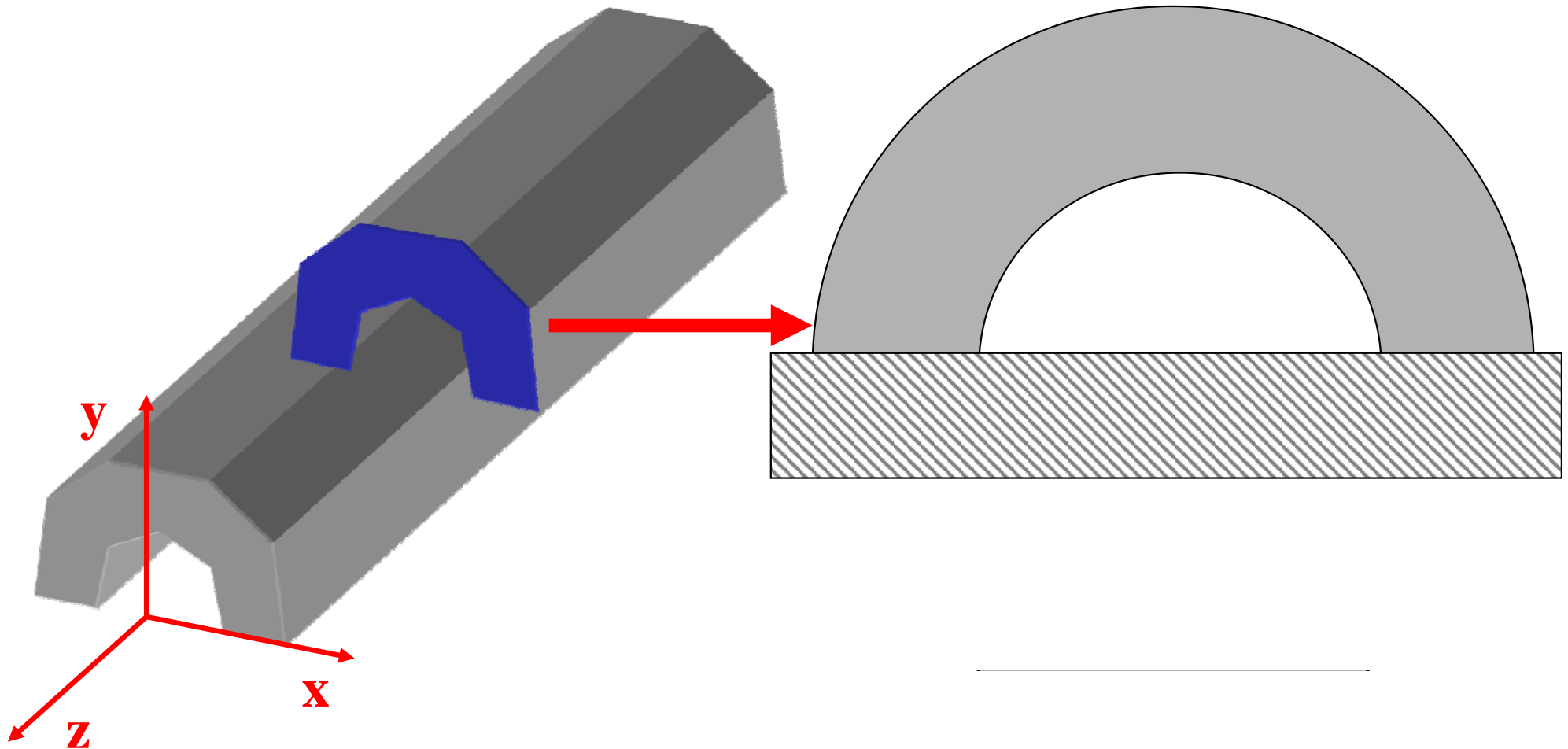
$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

$$\varepsilon_x, \varepsilon_y, \gamma_{xy} \neq 0$$

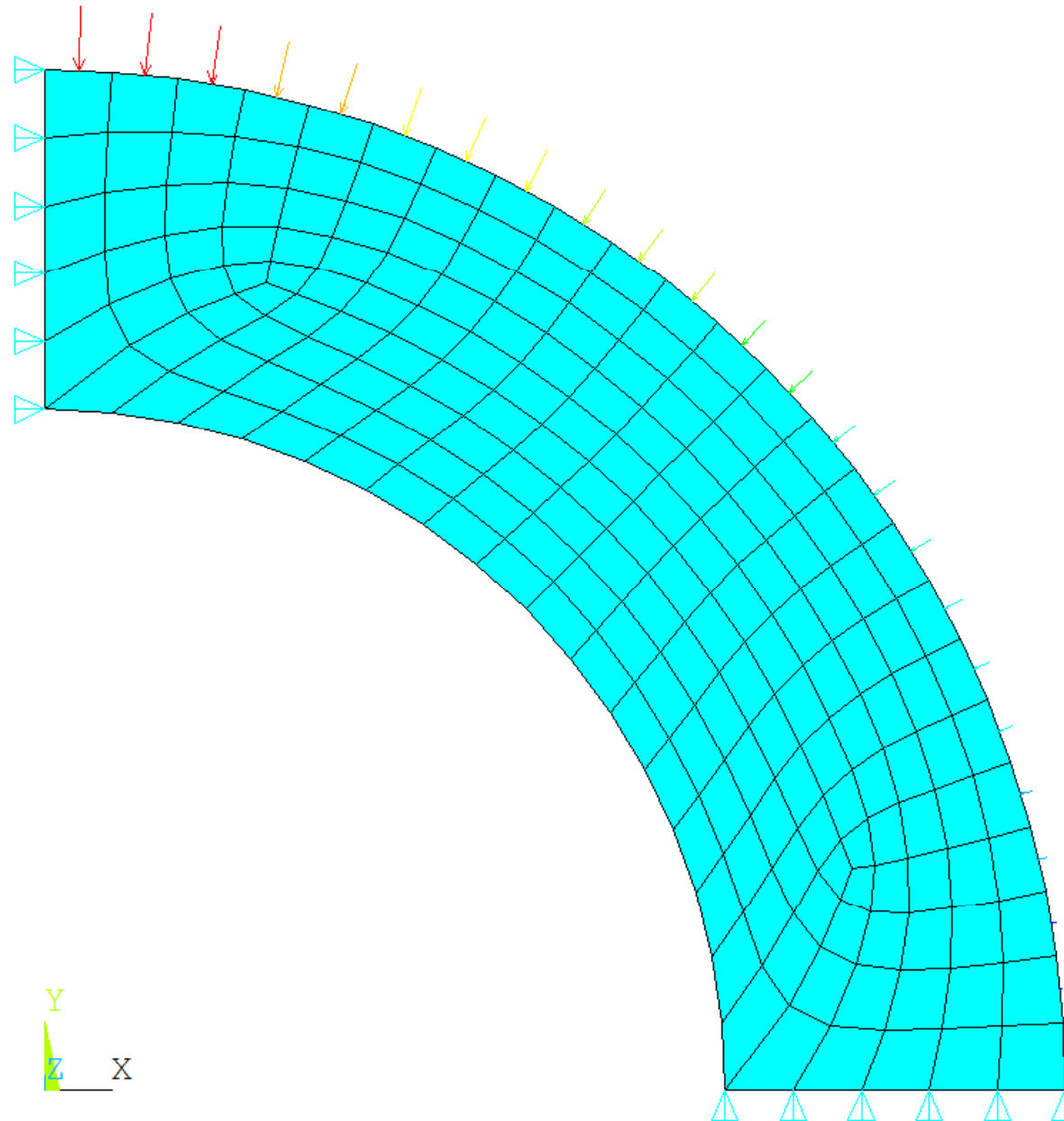


ELEMENTI PIANI/7

Il modello giace sul piano “x-y” e rappresenta una sezione, eseguita con un piano ortogonale all’asse z, della struttura. I carichi sono per unità di spessore.



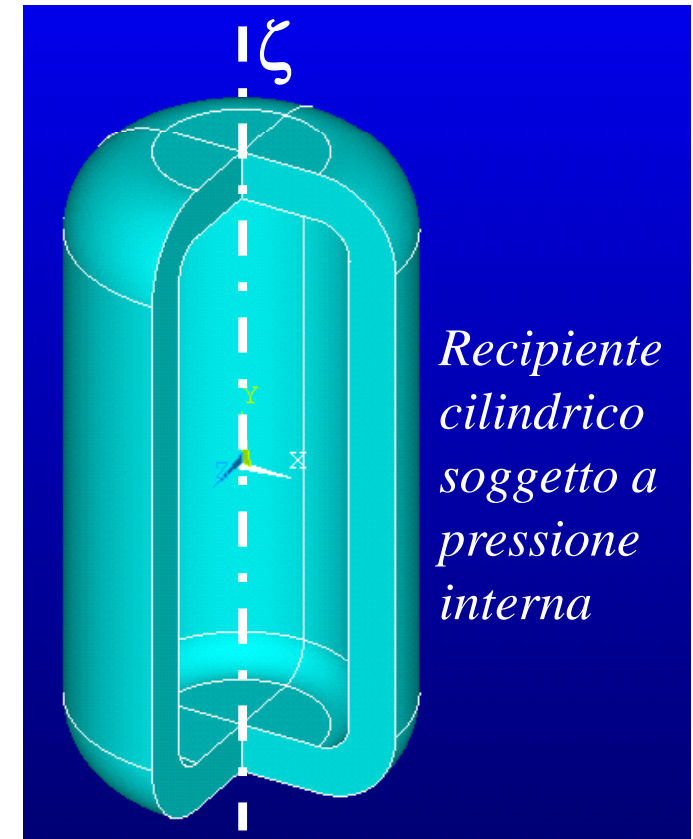
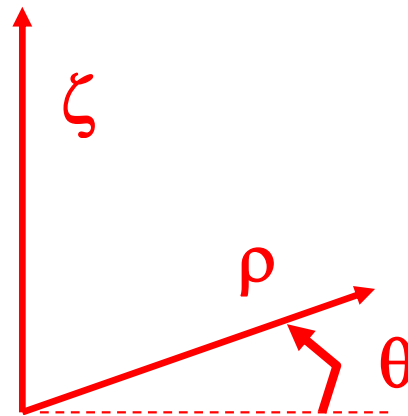
ELEMENTI PIANI/7



ELEMENTI PIANI/8

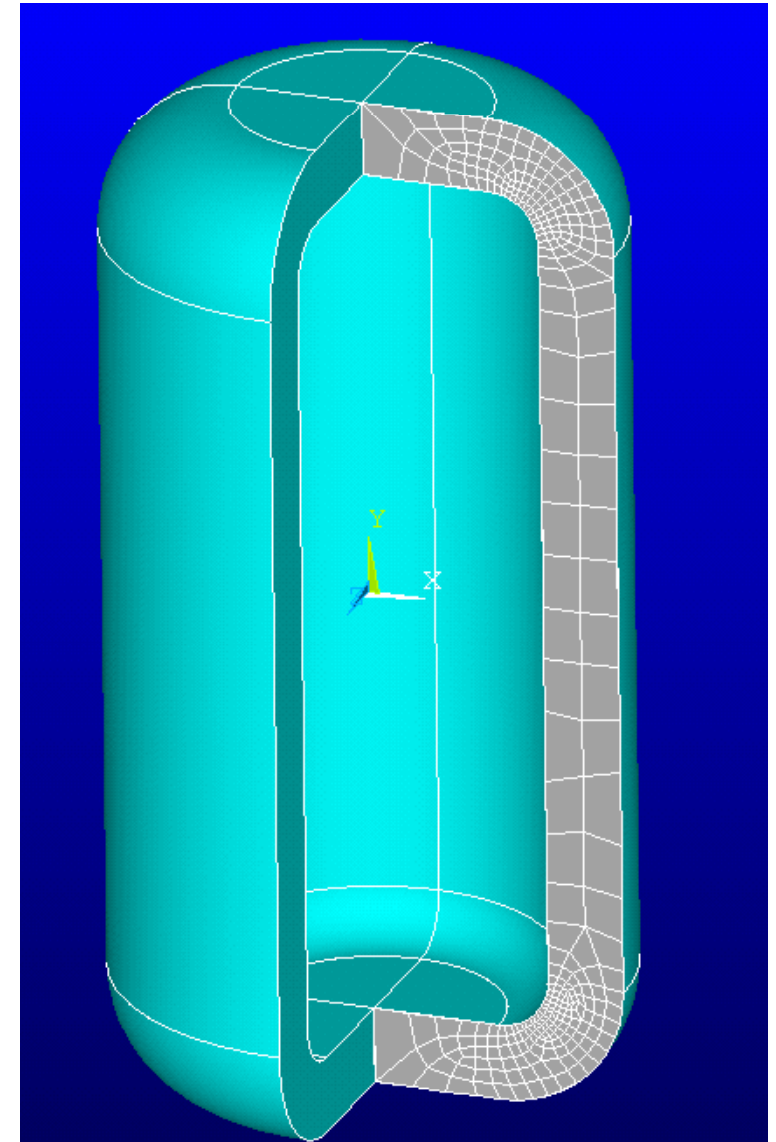
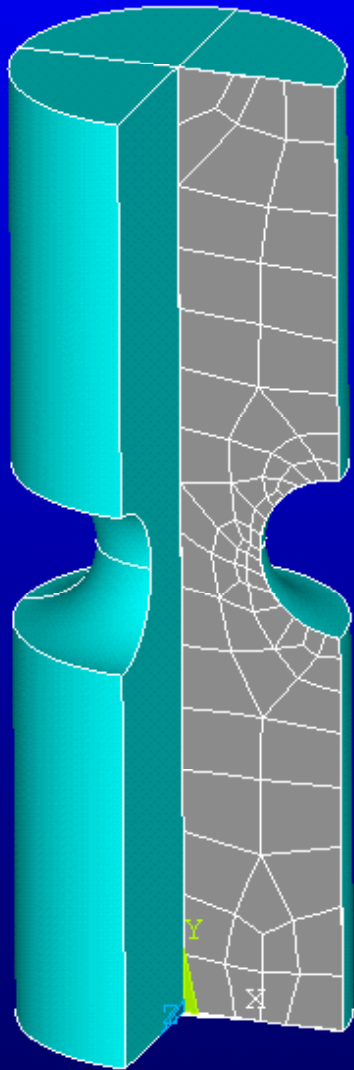
Stati assial-simmetrici

- si verificano in corpi di geometria assial-simmetrica (ottenibile per rotazione di una sezione attorno ad un asse fisso ζ) carichi con carichi che presentano lo stesso tipo di simmetria.
- fissato un SR cilindrico “ ρ, θ, ζ ”, per simmetria lo stato di tensione/deformazione risulta indipendente da θ e le componenti di spostamento in direzione circonferenziale (θ) risultano nulle: il problema può di conseguenza essere studiato come piano.

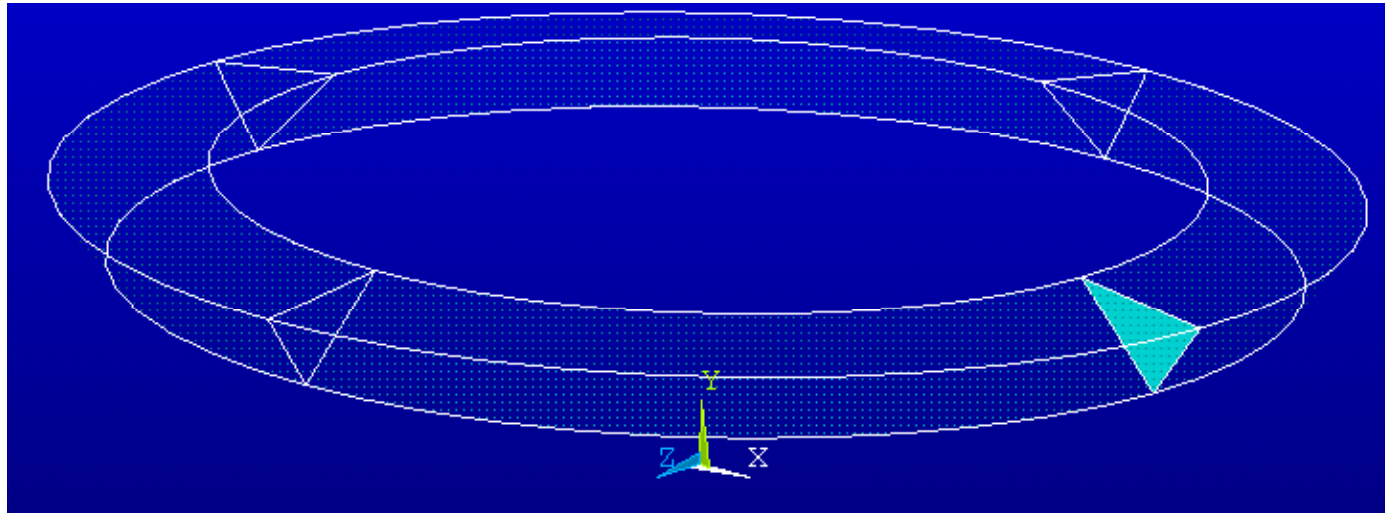


ELEMENTI PIANI/9

Il modello deve rappresentare una sezione del corpo fatta con un piano passante per l'asse di simmetria (in ANSYS, l'asse di simmetria e la direzione radiale devono coincidere rispettivamente con l'asse "Y" e l'asse "X" del SR cartesiano globale).



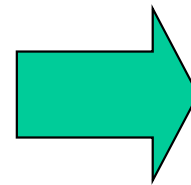
ELEMENTI PIANI/10



Volume
rappresentato
dall'elemento

$$\left\{ \begin{array}{l} \varepsilon_x = \frac{\partial v_x}{\partial x} \\ \varepsilon_y = \frac{\partial v_y}{\partial y} \\ \gamma_{xy} = \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \\ \varepsilon_\theta = \frac{v_x}{x} \end{array} \right.$$

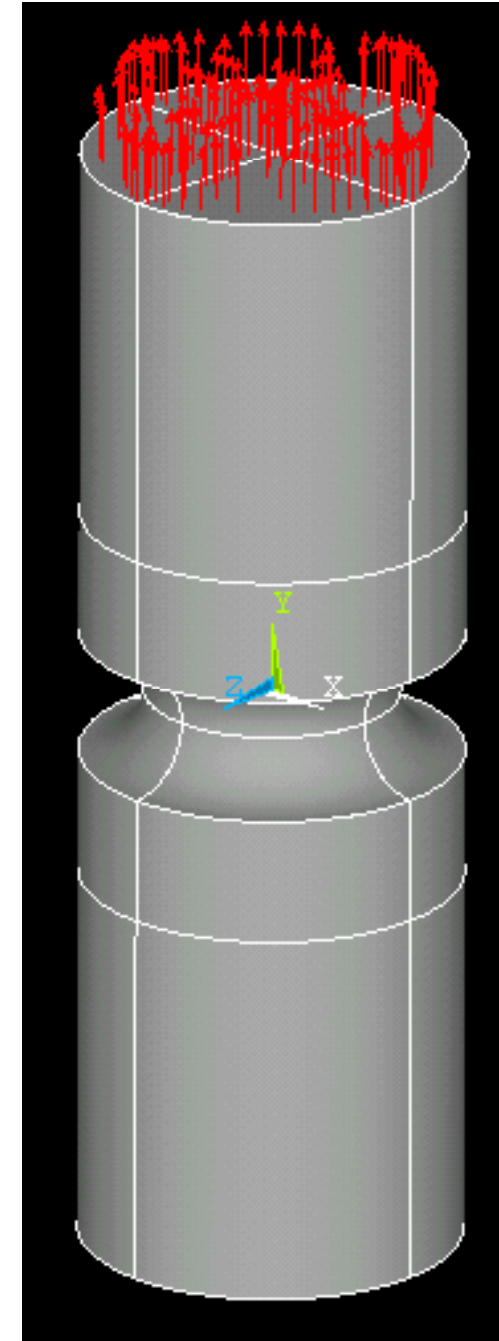
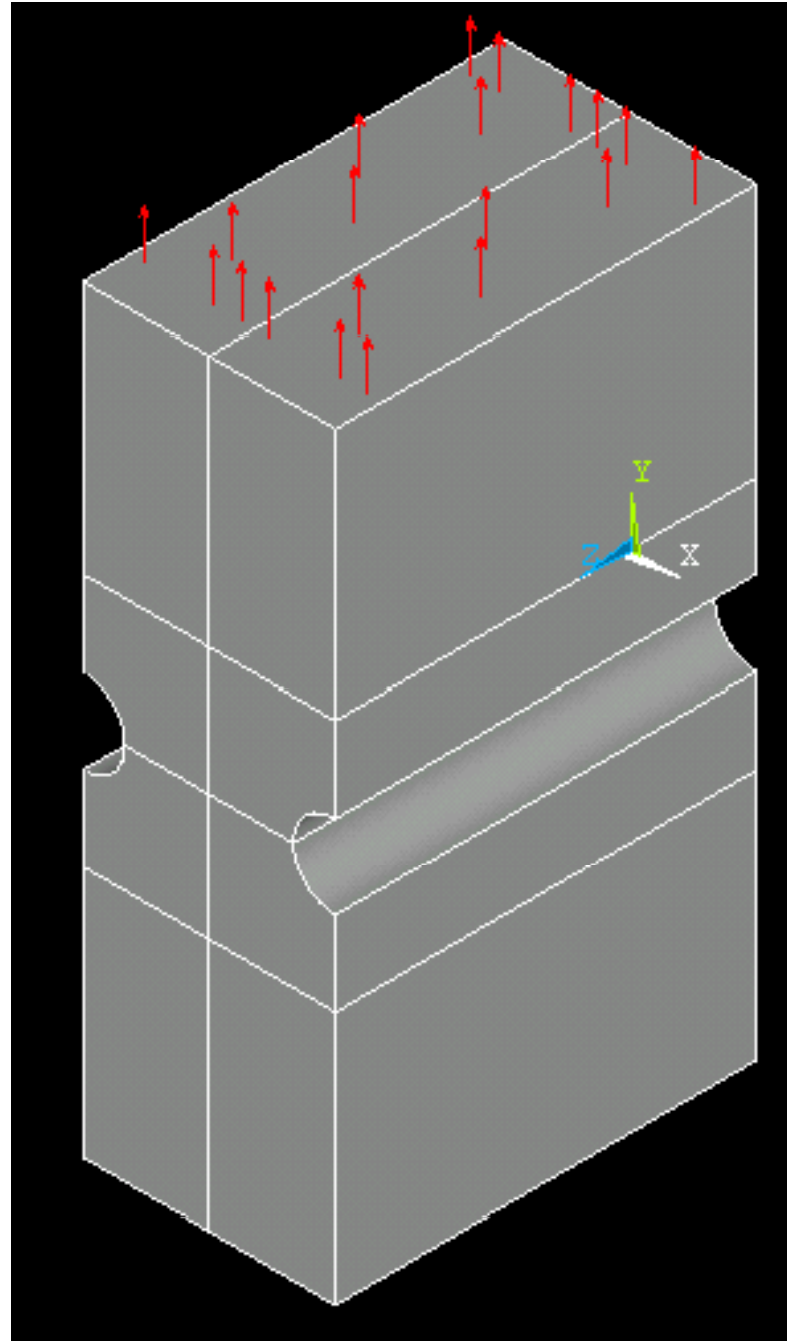
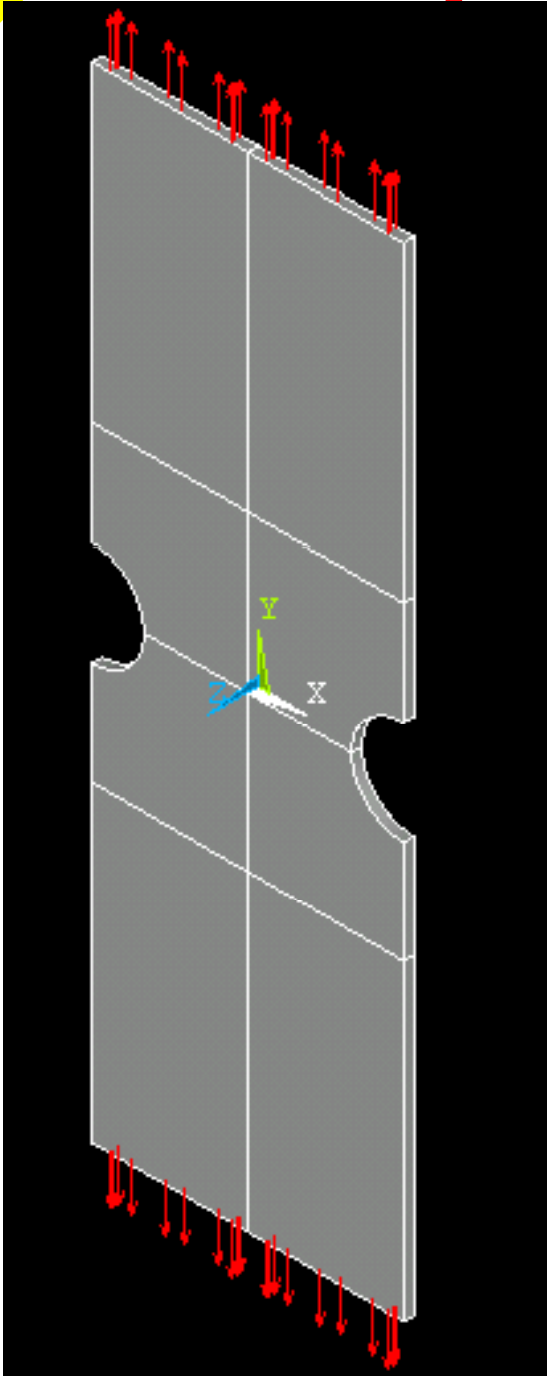
Rispetto al caso “plane stress” è necessario aggiungere una componente di deformazione/tensione



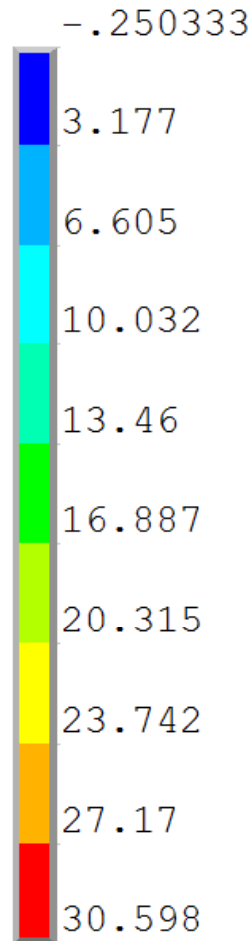
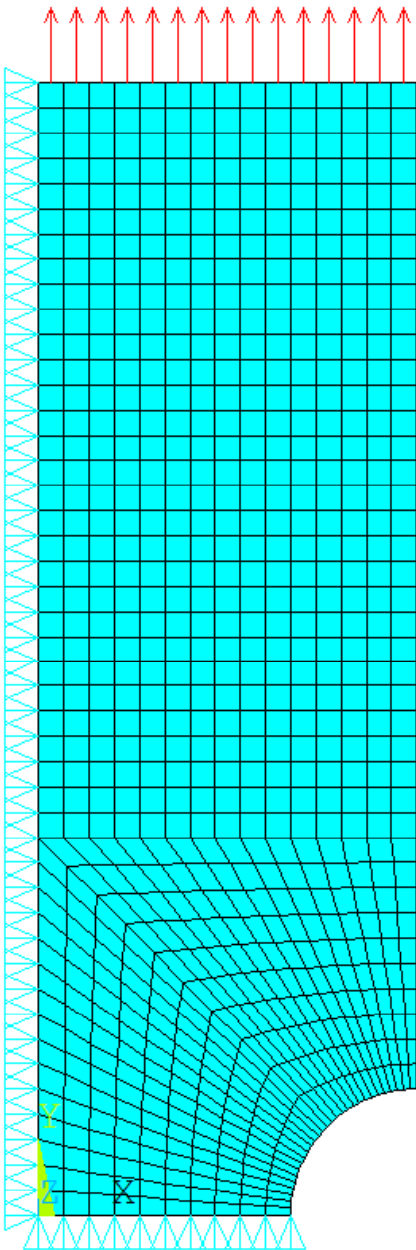
$$[L] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ \frac{1}{x} & 0 \end{bmatrix}$$

Esempio di applicazione

ELEMENTI PIANI/11

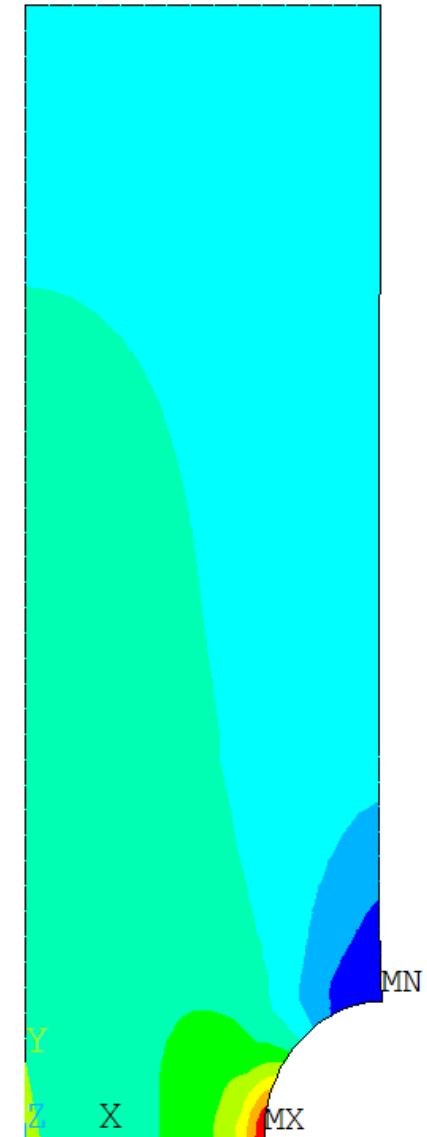


ELEMENTI PIANI/12



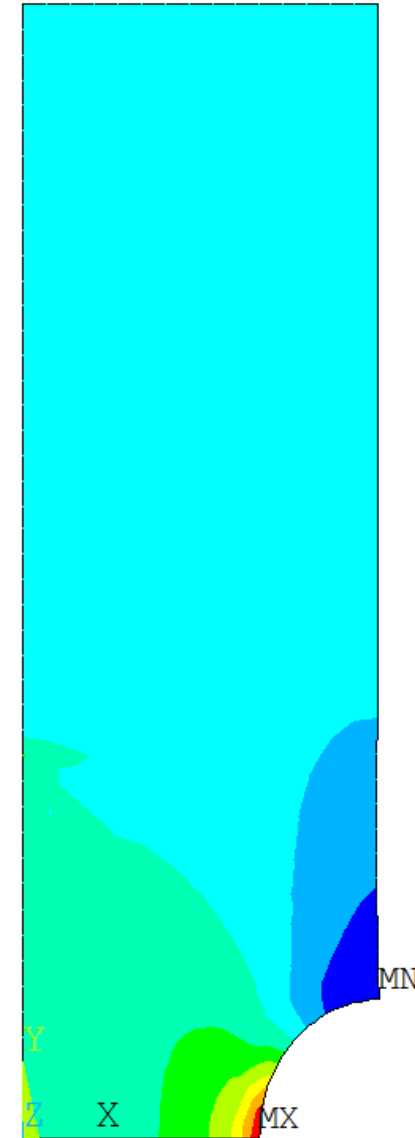
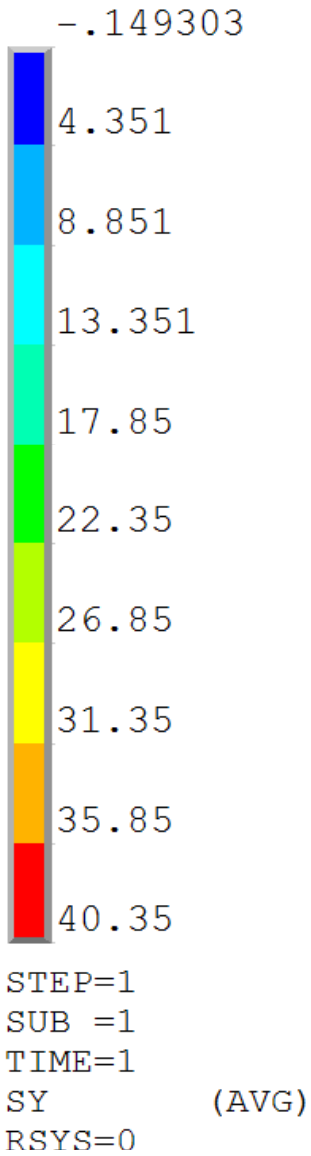
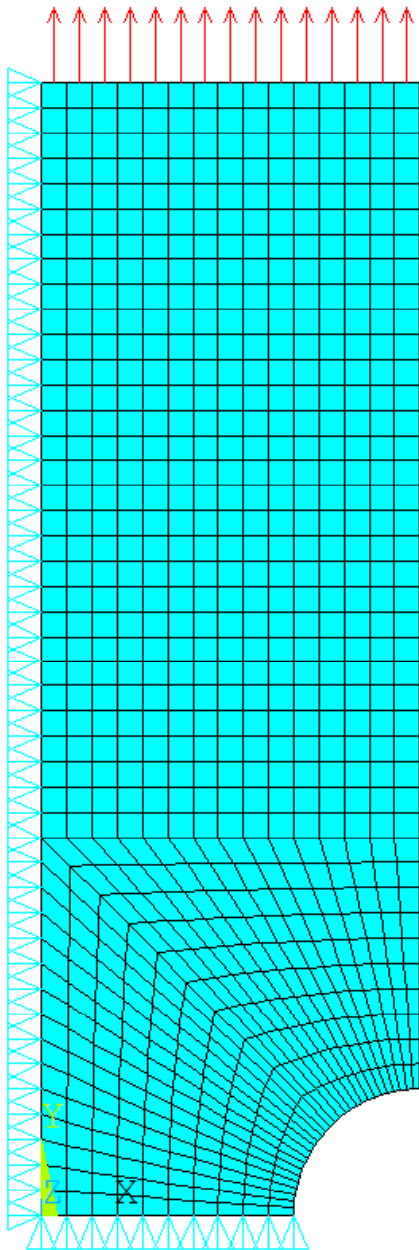
STEP=1
 SUB =1
 TIME=1
 SY (AVG)
 RSYS=0

STATO PIANO DI TENSIONE





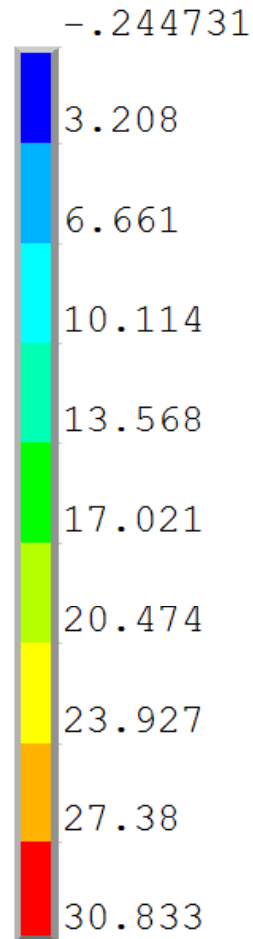
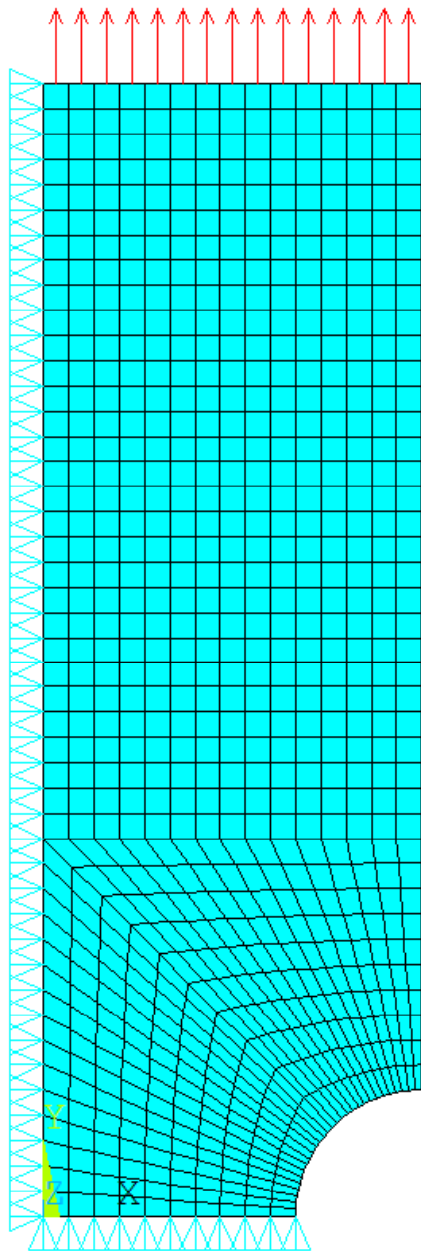
ELEMENTI PIANI/12



STATO DI TENSIONE ASSIALSIMMETRICO

File di comandi: ANALISI_PIANA_INTAGLIO.txt

ELEMENTI PIANI/12



STEP=1
 SUB =1
 TIME=1
 SY (AVG)
 RSYS=0

STATO PIANO DI DEFORMAZIONE

